

# Transverse Structure of Nucleon Parton Distributions

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Generalized parton distributions determine the angular momentum decomposition of the nucleon and the transverse distribution of partons in the nucleon. Additionally, in particular limits they reduce to form factors and ordinary parton distributions. I will review generalized parton distributions and present our full QCD lattice calculations of the quark helicity and orbital angular momentum in the nucleon, the asymptotic scaling ratio  $F_2/F_1$ , and the transverse distribution of quarks in the nucleon.

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[http://talks.drubryantrenner.org/uc\\_3-8-04.pdf](http://talks.drubryantrenner.org/uc_3-8-04.pdf)

## In Short

Generalized Parton Distributions determine

- 3D distribution of quarks in a mixed representation - 2 transverse coordinates and 1 longitudinal momentum
- decomposition of nucleon spin into quark helicity, quark orbital, and gluon contributions
- form factors and ordinary parton distributions

# Outline

I. Continuum Physics Review

II. Transverse Nucleon Structure

III. Lattice Calculation

IV. Our Results

# Continuum Physics Review

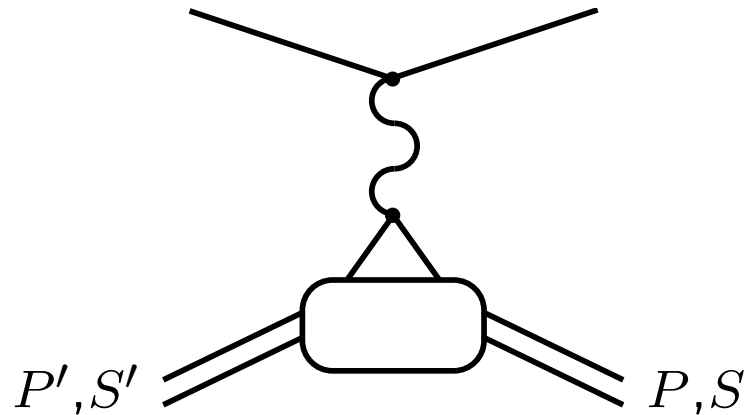
I. Form Factors

II. Parton Distributions

III. Generalized Parton Distributions

## Form Factors

- lepton-nucleon scattering,  $lN \rightarrow lN$ , elastic



$$\Delta = P' - P$$

$$t = \Delta^2$$

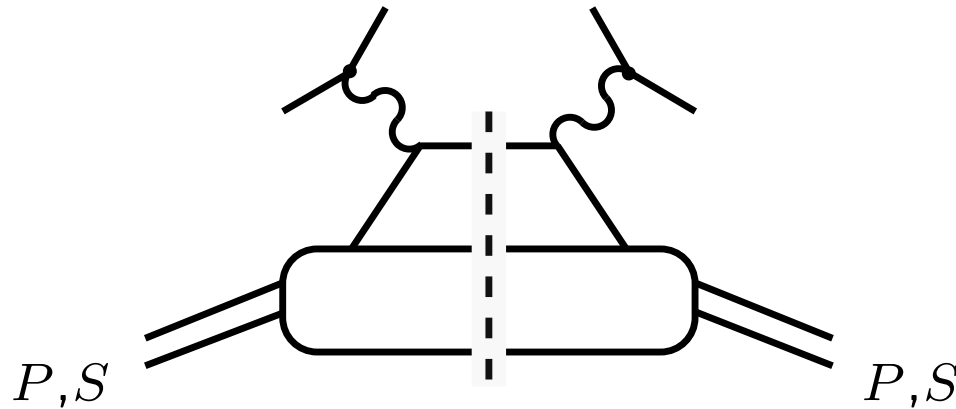
- off-forward matrix element of the electromagnetic current

$$\langle P', S' | J^\mu | P, S \rangle = \bar{U}(P', S') \left( \gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu} \Delta_\nu}{2m} F_2(t) \right) U(P, S)$$

- interpretation as Fourier transform of charge and current densities *in certain cases*
- magnetic moment, charge & current radii

# Parton Distributions

- deep inelastic scattering,  $lN \rightarrow lX$ , inclusive



$$P^+ = \frac{1}{2} (P^t + P^z)$$

$$y^- = \frac{1}{2} (y^t - y^z)$$

- forward matrix element of light-cone quark correlator

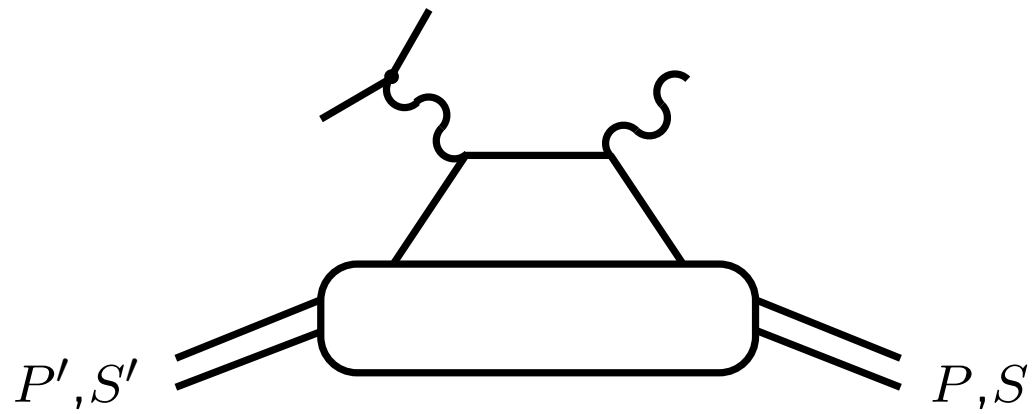
$$\mathcal{O}_q(x, \vec{b}_\perp) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \bar{q} \left( -\frac{y^-}{2}, \vec{b}_\perp \right) \gamma^+ q \left( \frac{y^-}{2}, \vec{b}_\perp \right)$$

$$\langle P, S | \mathcal{O}_q(x, \vec{0}_\perp) | P, S \rangle = q(x)$$

- longitudinal momentum distribution in the *infinite momentum frame*
- momentum fraction & spin fraction

## Generalized Parton Distributions

- deeply virtual Compton scattering,  $lN \rightarrow lN\gamma$ , exclusive



$$\bar{P} = \frac{1}{2} (P' + P)$$

$$\xi = -\frac{\Delta^+}{2\bar{P}^+}$$

- off-forward matrix element of light-cone quark correlator

$$\langle P', S' | \mathcal{O}_q(x, \vec{0}_\perp) | P, S \rangle =$$

$$\frac{1}{2\bar{P}^+} \bar{U}(P', S') \left( \gamma^+ H(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M} E(x, \xi, t) \right) U(P, S)$$

- meson production, Compton scattering

# Generalized Parton Distributions

- familiar limits: parton distributions & form factors

$$H_q(x, 0, 0) = q(x)$$

$$\sum_q e_q \int dx H_q(x, \xi, t) = F_1(t)$$

$$\sum_q e_q \int dx E_q(x, \xi, t) = F_2(t)$$

- quark angular momentum [1]

$$J_q = \frac{1}{2} \int dx x (H_q(x, 0, 0) + E_q(x, 0, 0))$$

- transverse quark distribution in *infinite momentum frame* [2]

$$\int d^2b_\perp e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} q(x, \vec{b}_\perp) = H_q(x, 0, -\vec{\Delta}_\perp^2)$$

[1] Ji hep-ph/9603249

[2] Burkardt hep-ph/0005108

## Transverse Structure

- I. Charge Distribution: Non-Relativistic Limit
- II. Transverse Distribution: Infinite Momentum Frame

## Charge Distribution

- wave packet

$$|\psi\rangle = \int \frac{d^3p}{(2\pi)^{3/2}} \frac{\psi(\vec{p})}{\sqrt{2E_{\vec{p}}}} |\vec{p}\rangle$$

- Fourier transform of charge distribution

$$\begin{aligned} \int d^3x e^{i\vec{q}\cdot\vec{x}} \langle\psi| J^0(\vec{x}) |\psi\rangle &= \\ &= \int d^3p \psi^*(\vec{k}) \psi(\vec{p}) \langle\vec{k}| J^0(0) |\vec{p}\rangle / \left(2\sqrt{E_{\vec{k}}E_{\vec{p}}}\right) \\ &= \int d^3p \psi^*(\vec{k}) \psi(\vec{p}) F(q^2) (E_{\vec{k}} + E_{\vec{p}}) / \left(2\sqrt{E_{\vec{k}}E_{\vec{p}}}\right) \end{aligned}$$

where  $\vec{k} = \vec{p} + \vec{q}$  &  $\langle\vec{k}| J^0(0) |\vec{p}\rangle = F(q^2) (E_{\vec{k}} + E_{\vec{p}})$

## Charge Distribution

- non-relativistic limit  $|\vec{q}| \ll M \quad E_{\vec{k}} \approx E_{\vec{p}} \quad q^2 \approx -\vec{q}^2$
- broad wave packet  $\psi(\vec{k}) \approx \psi(\vec{p})$

$$\begin{aligned} & \int d^3x e^{i\vec{q}\cdot\vec{x}} \langle \psi | J^0(\vec{x}) | \psi \rangle \\ &= \int d^3p \psi^*(\vec{k}) \psi(\vec{p}) F(q^2) \left( \frac{E_{\vec{k}} + E_{\vec{p}}}{2\sqrt{E_{\vec{k}}E_{\vec{p}}}} \right) \\ &\approx \int d^3p \psi^*(\vec{p}) \psi(\vec{p}) F(-\vec{q}^2) \\ &\approx F(-\vec{q}^2) \end{aligned}$$

## Charge Distribution

- separation of scales,  $L_{FF/WP}$  is the form factor/wave packet length scale

$$L_{FF} \sim \frac{1}{|\vec{q}|} \gg L_{WP} \gg \frac{1}{M} \quad \Rightarrow \quad L_{FF} \gg \frac{1}{M}$$

- hydrogen atom

$$L_{FF} = \frac{1}{\alpha m_e} = 7 \cdot 10^4 \text{ fm} \gg \frac{1}{M_H} = \frac{1}{M_P} = 0.2 \text{ fm}$$

- proton

$$L_{FF} = 0.87 \text{ fm} \approx \frac{1}{M_P} = 0.2 \text{ fm}$$

## Transverse Distribution

- wave packet

$$|\psi\rangle = \int \frac{d^2 p_\perp}{(2\pi)} \frac{\psi(\vec{p}_\perp)}{\sqrt{2E_{\vec{p}}}} |\vec{p}_\perp, P_z\rangle \quad \vec{p} = (\vec{p}_\perp, P_z)$$

- Fourier transform of transverse quark distribution

$$\mathcal{O}_q(x, \vec{b}_\perp) = \int \frac{dy^-}{4\pi} e^{ix\bar{P}^+ y^-} \bar{q}(-y^-/2, \vec{b}_\perp) \gamma^+ q(y^-/2, \vec{b}_\perp)$$

$$\int d^2 b_\perp e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \langle \psi | \mathcal{O}_q(x, \vec{b}_\perp) | \psi \rangle = \int d^2 p_\perp \psi^*(\vec{k}_\perp) \psi(\vec{p}_\perp) H_q(x, 0, \Delta^2) (2\sqrt{E_{\vec{k}}}\sqrt{E_{\vec{p}}})$$

$$\vec{k}_\perp = \vec{p}_\perp + \vec{q}_\perp \quad k_z = p_z = P_z \quad \langle \vec{k} | \mathcal{O}_q(x, \vec{0}_\perp) | \vec{p} \rangle = H_q(x, 0, \Delta^2)$$

## Transverse Distribution

- infinite momentum limit  $|\vec{q}_\perp| \ll P_z$   $E_{\vec{k}} \approx E_{\vec{p}}$   $\Delta^2 \approx -\vec{\Delta}_\perp^2$
- broad wave packet  $\psi(\vec{k}_\perp) \approx \psi(\vec{p}_\perp)$

$$\begin{aligned} & \int d^2 b_\perp e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \langle \psi | \mathcal{O}_q(x, \vec{b}_\perp) | \psi \rangle \\ &= \int d^2 p_\perp \psi^*(\vec{k}_\perp) \psi(\vec{p}_\perp) H_q(x, 0, \Delta^2) / (2\sqrt{E_{\vec{k}}} \sqrt{E_{\vec{p}}}) \\ &\approx \int d^2 p_\perp \psi^*(\vec{p}_\perp) \psi(\vec{p}_\perp) H_q(x, 0, -\vec{\Delta}_\perp^2) / 2P_z \\ &\approx H_q(x, 0, -\vec{\Delta}_\perp^2) \end{aligned}$$

## Transverse Distribution

- separation of scales,  $L_{TD}/L_{WP}$  is the transverse distribution/wave packet length scale

$$L_{TD} \sim \frac{1}{|\vec{\Delta}_{\perp}|} \gg L_{WP} \gg \frac{1}{\sqrt{M^2 + P_z^2}} \quad \Rightarrow \quad L_{TD} \gg \frac{1}{\sqrt{M^2 + P_z^2}} \rightarrow 0$$

- transverse quark distribution

$$\int d^2b_{\perp} e^{i\vec{\Delta}_{\perp} \cdot \vec{b}_{\perp}} q(x, \vec{b}_{\perp}) = H_q(x, 0, -\vec{\Delta}_{\perp}^2)$$

## Lattice Calculation

- I. Moments of Generalized Parton Distributions
- II. Generalized Form Factors
- III. Matrix Elements from Lattice QCD

## Moments of Generalized Parton Distributions

- generalized parton distributions

$$\langle P', S' | \mathcal{O}_q(x) | P, S \rangle = \frac{1}{2\bar{P}^+} \bar{U}(P', S') \left( \gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M} E_q(x, \xi, t) \right) U(P, S)$$

$$\mathcal{O}_q(x, \vec{b}_\perp) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \bar{q} \left( -\frac{y^-}{2}, \vec{b}_\perp \right) \gamma^+ q \left( \frac{y^-}{2}, \vec{b}_\perp \right)$$

- light-cone expansion & tower of twist 2 operators

$$\mathcal{O}_q^{\mu_1 \mu_2 \dots \mu_n} = \bar{q} \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} q$$

- moments of generalized parton distributions

$$\langle P' | \mathcal{O}^{\mu_1 \mu_2 \dots \mu_n} | P \rangle \sim \int dx x^{n-1} H_q(x, \xi, t) \quad \& \quad \int dx x^{n-1} E_q(x, \xi, t)$$

# Generalized Form Factors

- $n + 1$  generalized form factors

$$\langle P' | \mathcal{O}_q^{\mu_1 \mu_2 \dots \mu_n} | P \rangle \sim A_{ni}^q(t), B_{ni}^q(t), C_n^q(t)$$

- moments of parton distributions

$$\langle x^{n-1} \rangle_q = A_{n0}^q(0)$$

- form factors

$$F_1(t) = \sum_q e_q A_{10}^q(t) \quad F_2(t) = \sum_q e_q B_{10}^q(t)$$

- quark angular momentum

$$J_q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0))$$

## Generalized Form Factors

- moments of generalized parton distributions

$$\int_{-1}^1 dx x^{n-1} H_q(x, \xi, t) = \sum_{i=0}^{[(n-1)/2]} A_{n,2i}^q(t) (-2\xi)^{2i} + \text{mod}(n+1, 2) C_n^q(t) (-2\xi)^n$$

$$\int_{-1}^1 dx x^{n-1} E_q(x, \xi, t) = \sum_{i=0}^{[(n-1)/2]} B_{n,2i}^q(t) (-2\xi)^{2i} - \text{mod}(n+1, 2) C_n^q(t) (-2\xi)^n$$

- transverse momentum transfer,  $\xi \rightarrow 0$

$$\int_{-1}^1 dx x^{n-1} H_q(x, 0, t) = A_{n0}^q(t)$$

$$\int_{-1}^1 dx x^{n-1} E_q(x, 0, t) = B_{n0}^q(t)$$

## Lattice Correlation Functions

- correlation function of  $2N$  fermion fields  $(\psi, \bar{\psi})$  and  $M$  gauge links ( $U = e^{iagA}$ )

$$\begin{aligned} \langle \psi_1 \cdots \psi_N \bar{\psi}_1 \cdots \bar{\psi}_N U_1 \cdots U_M \rangle &= \\ &= \int DU \int D\psi D\bar{\psi} e^{-S_G[U]} e^{-\bar{\psi} M[U] \psi} \psi_1 \cdots \psi_N \bar{\psi}_1 \cdots \bar{\psi}_N U_1 \cdots U_M \\ &= \int DU e^{-S_G[U]} \det^{N_f}(M[U]) U_1 \cdots U_M \sum_{\pi} (-1)^{\pi} M[U]_{1\pi_1}^{-1} \cdots M[U]_{N\pi_N}^{-1} \end{aligned}$$

- numerical integration

$$\int DU e^{-S_G[U]} \det^{N_f}(M[U]) F[U] = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N F[U^i]$$

## Matrix Elements from Lattice Correlation Functions

- nucleon interpolating field with gauge invariant smearing

$$J^\alpha = U_a^\alpha (U_b^T C \gamma^5 D_c) \epsilon_{abc}$$

- lattice ratios: fixed source (src), momentum projection at operator (op), sink (snk)

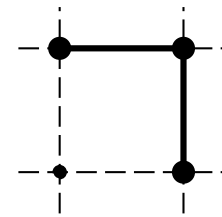
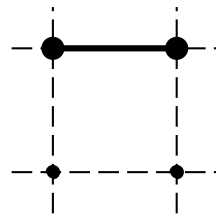
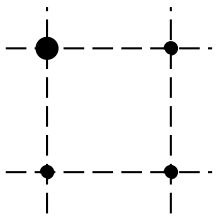
$$\langle p', s' | \mathcal{O} | p, s \rangle \sim \frac{\langle J(x_{\text{snk}}) \mathcal{O}(x_{\text{op}}) \bar{J}(x_{\text{src}}) \rangle}{\langle J(x_{\text{snk}}) \bar{J}(x_{\text{src}}) \rangle}$$

- quantum mechanical matrix elements are given by appropriate ratios of lattice correlation functions

## Building Blocks

- two quark operators can be written in terms of a basic set of building blocks

$$\bar{q}(x) \Gamma_i q(x) \quad \bar{q}(x + \hat{\mu}) \Gamma_i U_\mu^\dagger(x) q(x) \quad \bar{q}(x + \hat{\nu} + \hat{\mu}) \Gamma_i U_\nu^\dagger(x + \hat{\mu}) U_\mu^\dagger(x) q(x)$$



- $\Gamma_i$  ( $i = 1, \dots, 16$ ) denote a complete basis of  $4 \times 4$  Dirac spin matrices
- calculate upto 3 gauge link insertions:  $U_{\mu_n}^\dagger(x + \hat{\mu}_{n-1} + \dots + \hat{\mu}_1) \dots U_{\mu_1}^\dagger(x)$
- calculate a broad range of momentum transfers  $\vec{q}$

## Overdetermined Set of Lattice Observables

- $i$  labels all combinations of operator indices  $(\mu_1, \dots, \mu_n)$  and momenta  $P', P$

$$\mathcal{O}_i^{\overline{\text{MS}}} = \langle P' | \mathcal{O}^{\mu_1 \dots \mu_n} | P \rangle$$

- expand  $\mathcal{O}_i^{\overline{\text{MS}}}$  in terms of generalized form factors

$$\mathcal{O}_i^{\overline{\text{MS}}} = \sum_{\alpha=1}^{n+1} A_{i\alpha} F_\alpha(t)$$

- match  $\mathcal{O}_i^{\overline{\text{MS}}}$  to  $\mathcal{O}_i^{\text{lat}}$  with one loop matching coefficients  $Z_{ij}$

$$\mathcal{O}_i^{\overline{\text{MS}}} = \sum_j Z_{ij} \mathcal{O}_j^{\text{lat}}$$

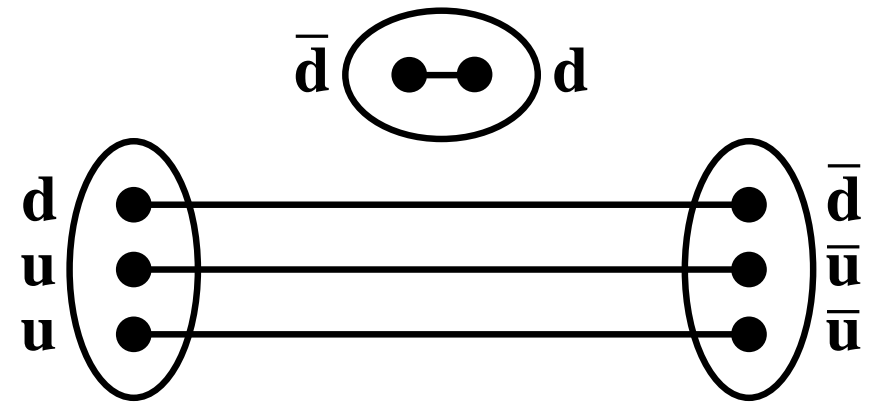
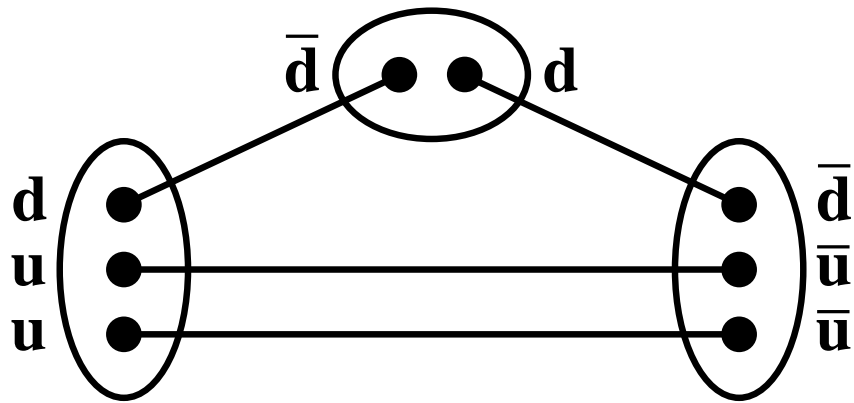
- construct the appropriate ratio of lattice correlation functions  $R_i$

$$R_i = \sum_j B_{ij} \mathcal{O}_j^{\text{lat}}$$

- overdetermined set of observables

$$R_i = \sum_{\alpha=1}^{n+1} C_{i\alpha} F_\alpha(t) \quad \chi^2 = \sum_i \left( \frac{\sum_{\alpha=1}^{n+1} C_{i\alpha} F_\alpha(t) - R_i}{\sigma_i} \right)^2$$

## Connected & Disconnected Diagrams



- connected diagrams require  $18 = 3 \cdot 2 \cdot 3$  Dirac matrix inversions per gauge field, determines non-singlet matrix elements ( $u - d$ ) for  $m_u = m_d$
- disconnected diagrams require  $\# \cdot (t_{\text{sink}} - t_{\text{source}}) \cdot V$  Dirac matrix inversions per gauge field, necessary for singlet matrix elements ( $u + d$ )
- other methods: truncated eigenmode approach, stochastic estimation, wall source technique

## Our Results

I. Simulation Parameters

II. Form Factors

III. Quark Angular Momenta

IV. Transverse Quark Distributions

## Simulation Parameters

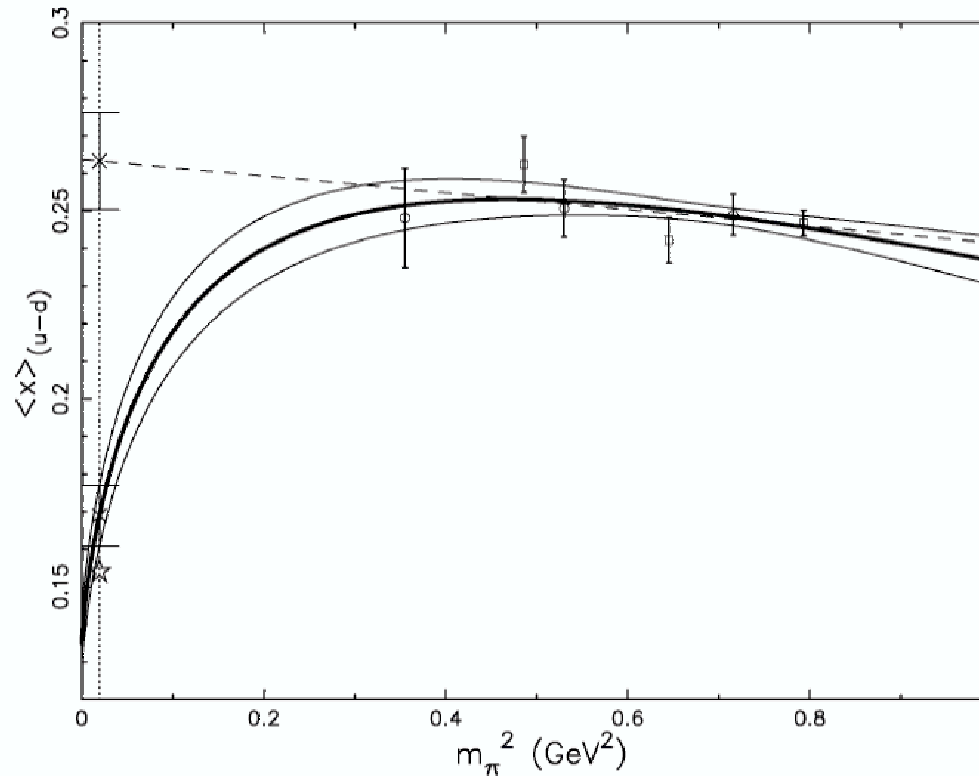
- SESAM gauge fields
- $N_F = 2$  Wilson fermions
- $a = 0.092$  fm,  $L = 1.48$  fm,  $16^3 \times 32$
- O(200) gluon configurations
- $M_\pi = 744, 831, 897$  MeV

## Chiral Extrapolations

- leading order  $\chi$ PT & heavy quark limit

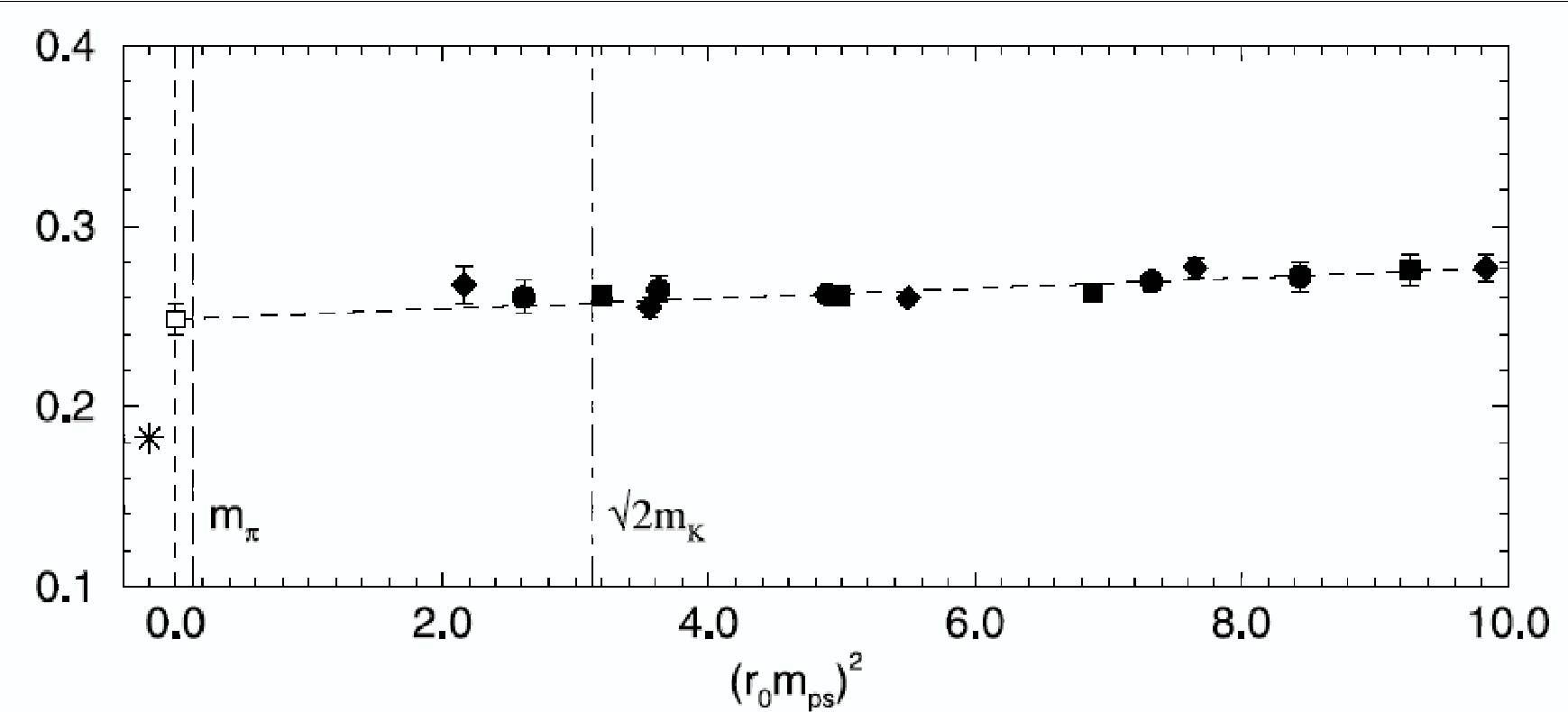
$$\langle x^n \rangle_{u-d} = a_n \left( 1 - \frac{(3g_A^2 + 1)m_\pi^2}{(4\pi f_\pi)^2} \ln \left( \frac{m_\pi^2}{m_\pi^2 + \mu^2} \right) \right) + b_n m_\pi^2$$

- $\langle x \rangle$  for SESAM full QCD



# Heavy Pion World

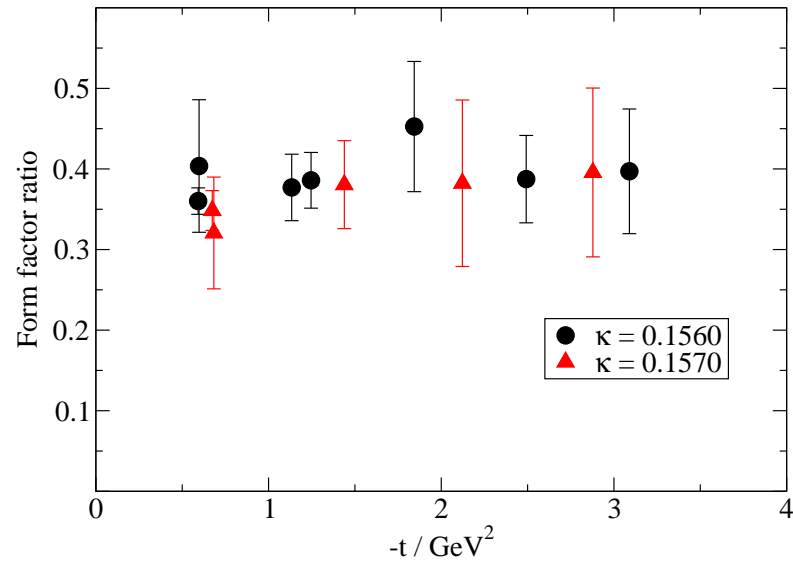
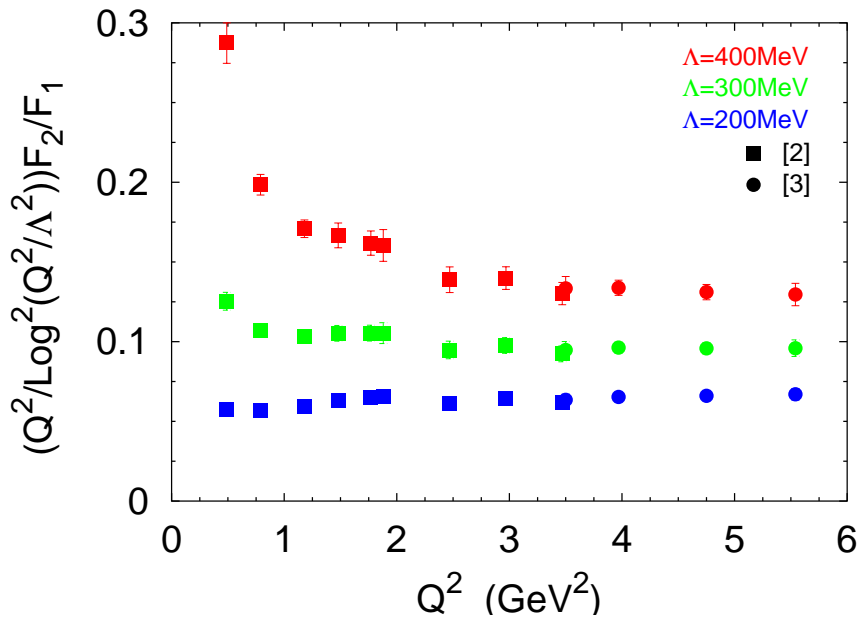
- linear  $m_\pi^2$  regime
- $\langle x \rangle$  for QCDSF quenched QCD



# Form Factor Ratios

- perturbative QCD & quark counting rules

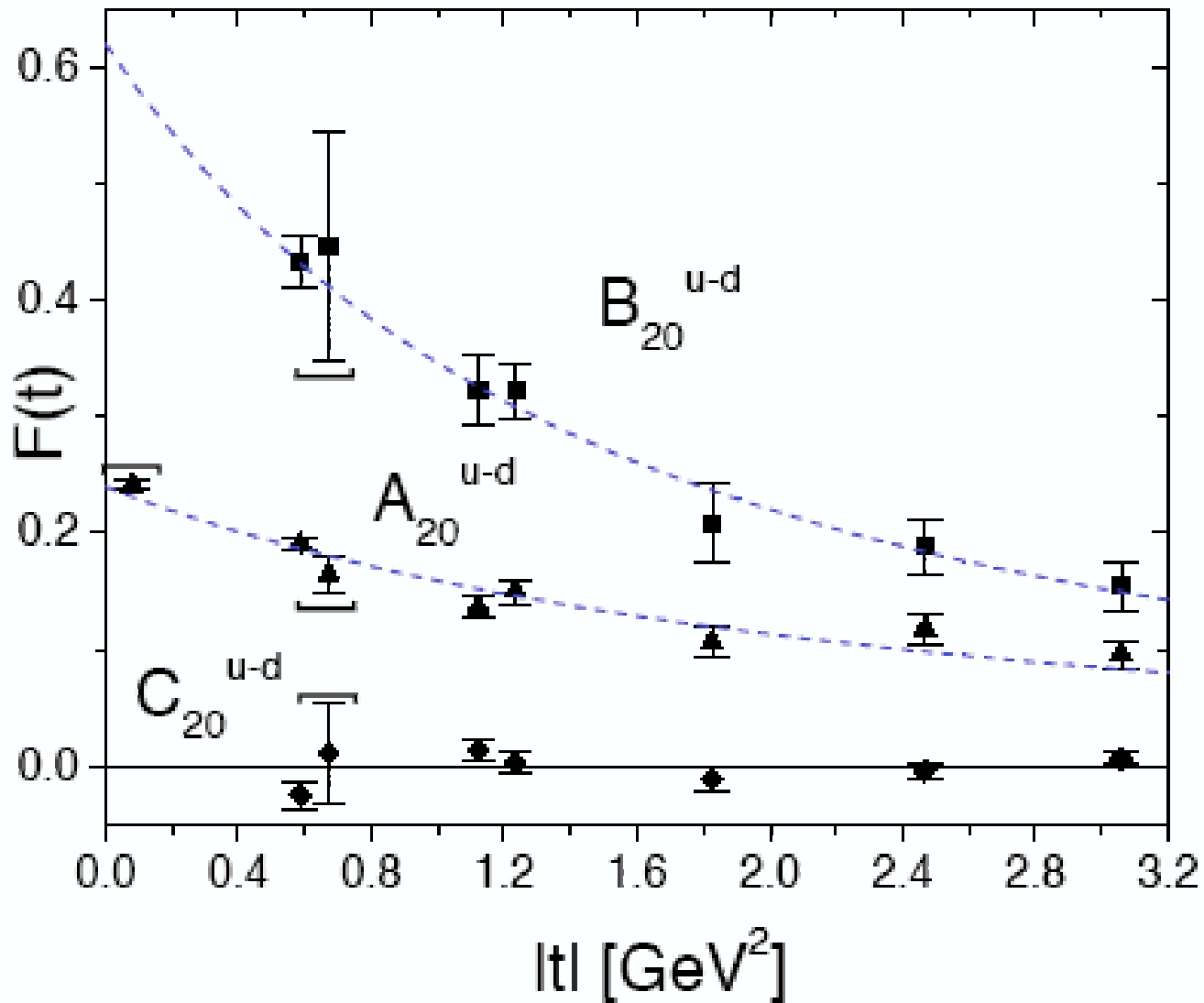
$$\frac{Q^2 F_2(Q^2)}{\log^2(Q^2/\Lambda^2) F_1(Q^2)} \sim \text{constant}$$



- note normalization difference: compare right plot with 1.79 times left plot

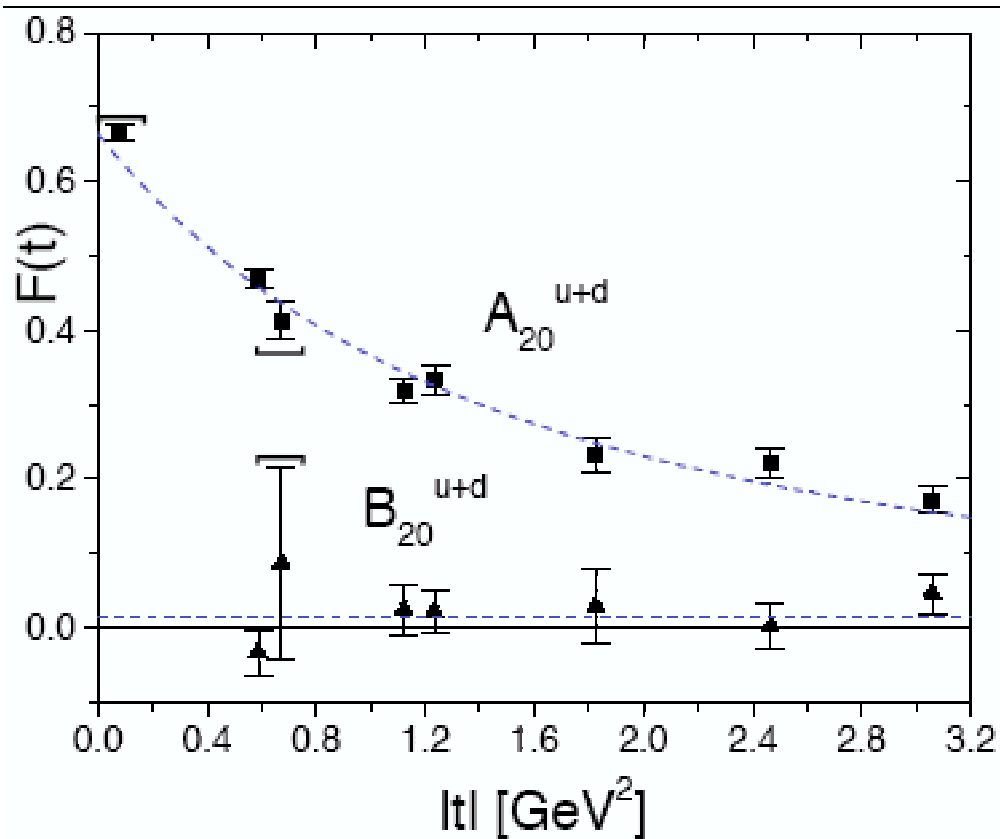
# Generalized Form Factors

- $\bar{\psi}\gamma^{\{\mu}D^{\nu\}}\psi$  has 3 GFFs:  $A_{20}$ ,  $B_{20}$ ,  $C_2$



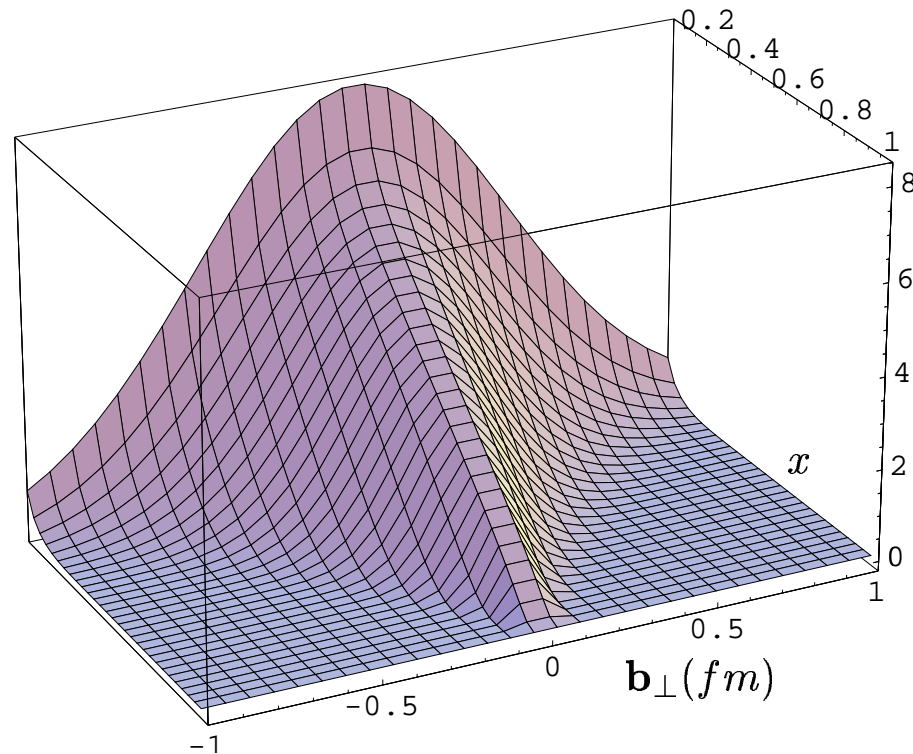
## Quark Angular Momentum: $m_\pi = 897\text{MeV}$

- $\frac{1}{2}\Delta\Sigma_{u+d} = \frac{1}{2}\tilde{A}_{10}^{u+d}(0) = \frac{1}{2}(\langle 1 \rangle_{\Delta u} + \langle 1 \rangle_{\Delta d}) \sim \frac{1}{2}0.682(18)$
- $J_{u+d} = \frac{1}{2}(A_{20}^{u+d}(0) + B_{20}^{u+d}(0)) \sim \frac{1}{2}0.675(7)$
- $L_{u+d} = J_{u+d} - \frac{1}{2}\Delta\Sigma_{u+d} \sim 0 \Rightarrow J_g = \frac{1}{2}0.32$



# Transverse Structure

- 3D quark distribution:  $q(x, \vec{b}_\perp)$
- $x$  is longitudinal momentum fraction
- $\vec{b}_\perp$  is transverse displacement of active parton relative to nucleon center of longitudinal momentum
- simple model calculation



## Transverse Structure

$$H_q(x, 0, -\vec{\Delta}_\perp^2) = \int d^2b_\perp e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} q(x, \vec{b}_\perp)$$

$$A_{n0}^q(t) = \int_{-1}^1 dx x^{n-1} H_q(x, 0, t)$$

- at  $x = 1$  a single quark carries all the momentum

$$\lim_{x \rightarrow 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp) \quad H_q(1, 0, t) = \text{constant}$$

- higher moments  $A_{n0}^q$  weight  $x \sim 1$  more heavily

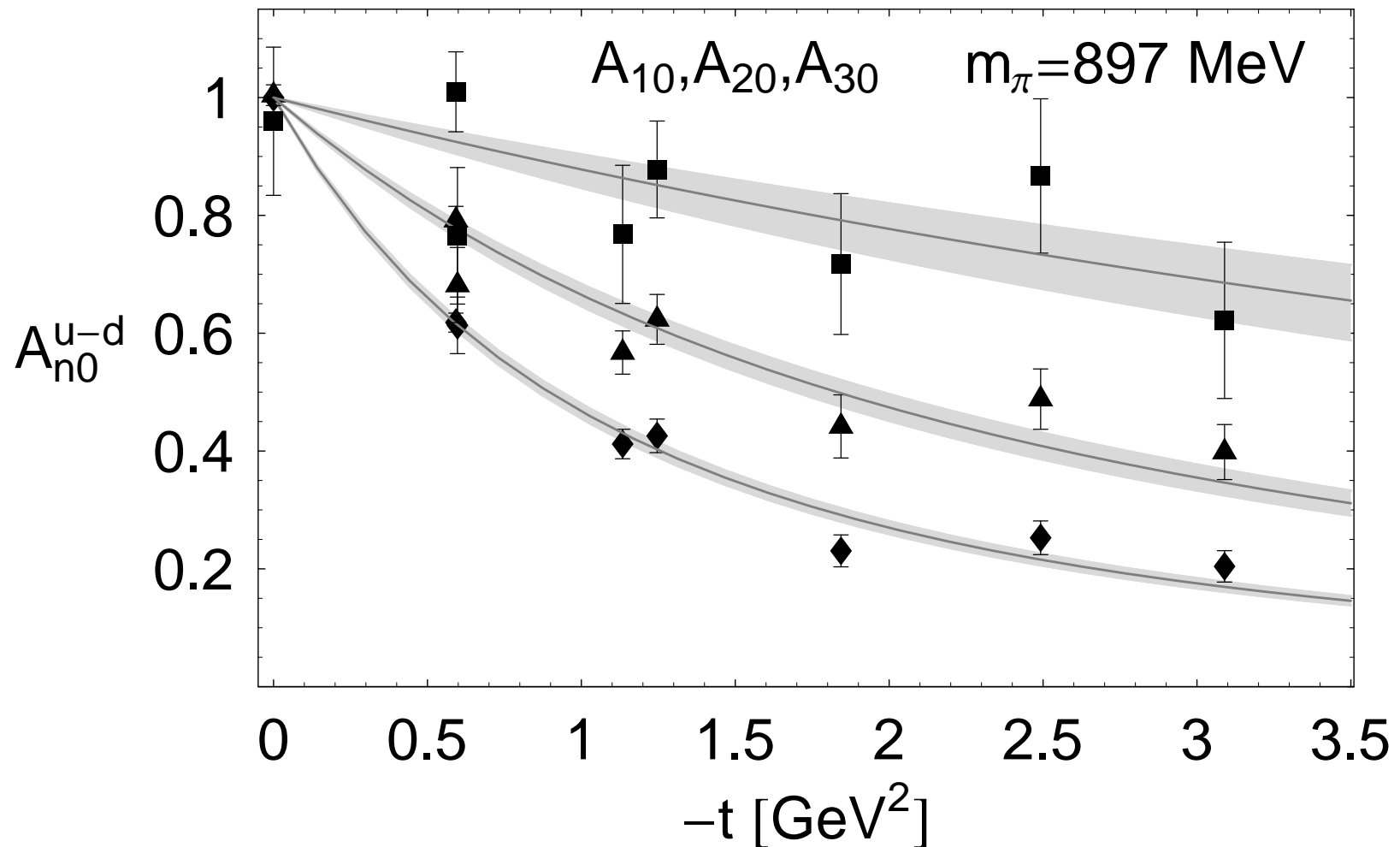
$$\lim_{n \rightarrow \infty} A_{n0}^q(t) \propto H_q(1, 0, t)$$

- slopes of  $A_{n0}^q$  should decrease as  $n$  increases

- $A_{10}, A_{30}, \tilde{A}_{20}$  measure  $q - \bar{q}$  &  $\tilde{A}_{10}, \tilde{A}_{30}, A_{20}$  measure  $q + \bar{q}$

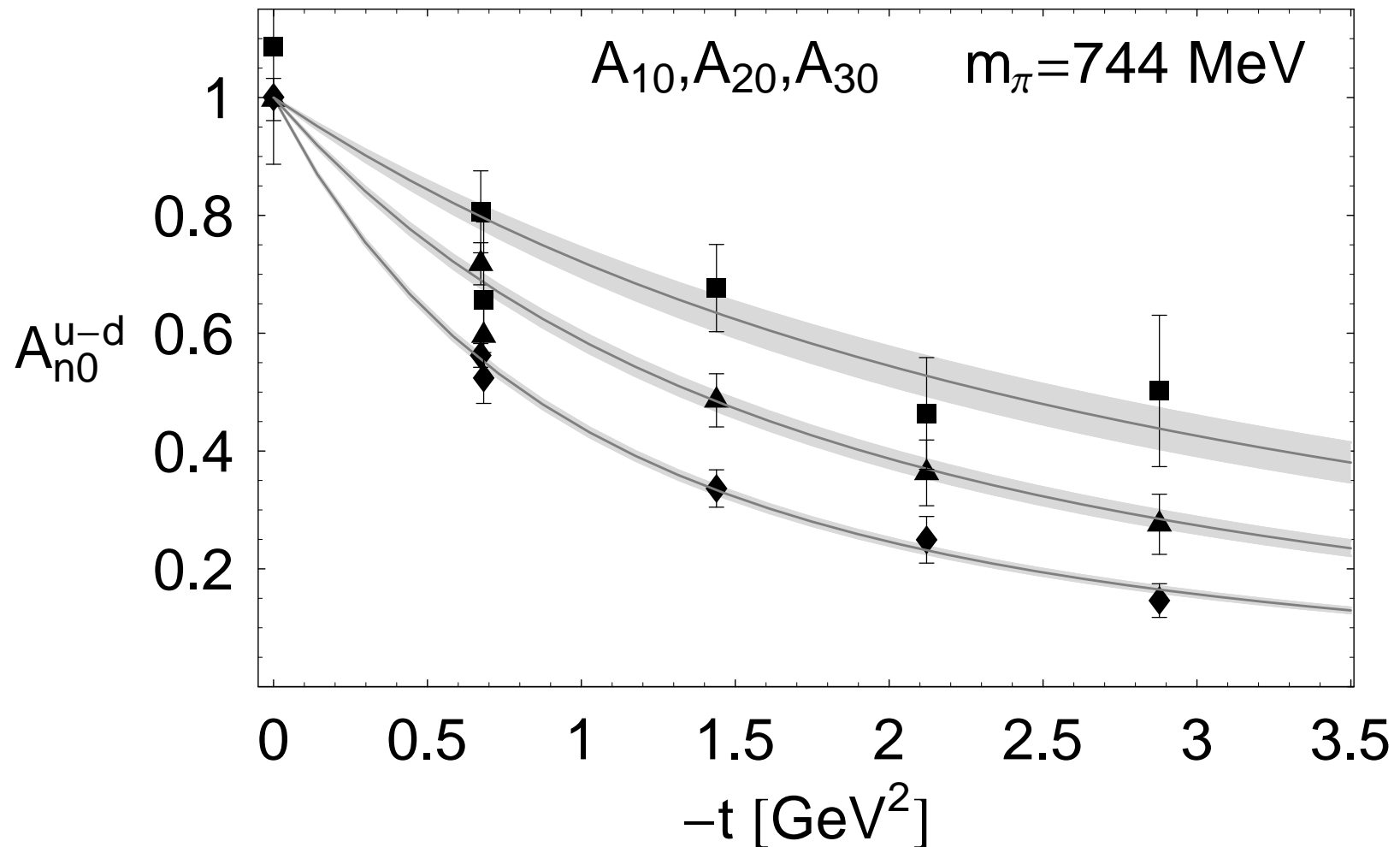
## Transverse Structure: $m_\pi = 897\text{MeV}$

- slope of  $A_{10}^{u-d} = -0.93 \pm 0.04 \text{ (GeV)}^{-2}$
- slope of  $A_{30}^{u-d} = -0.13 \pm 0.03 \text{ (GeV)}^{-2}$  (factor of 7)



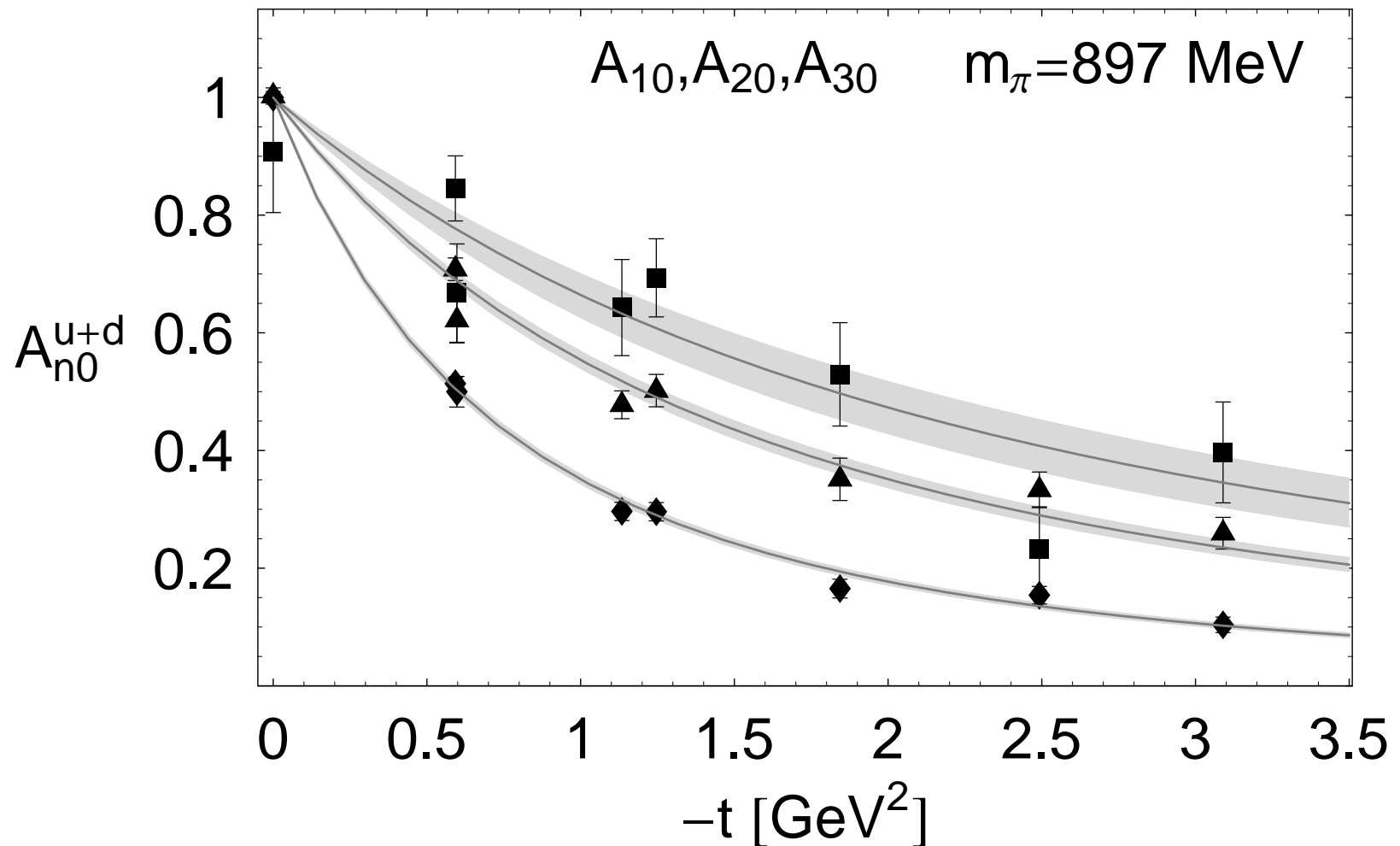
## Transverse Structure: $m_\pi = 744\text{MeV}$

- slope of  $A_{10}^{u-d} = -1.02 \pm 0.03 \text{ (GeV)}^{-2}$
- slope of  $A_{30}^{u-d} = -0.36 \pm 0.04 \text{ (GeV)}^{-2}$  (factor of 3)



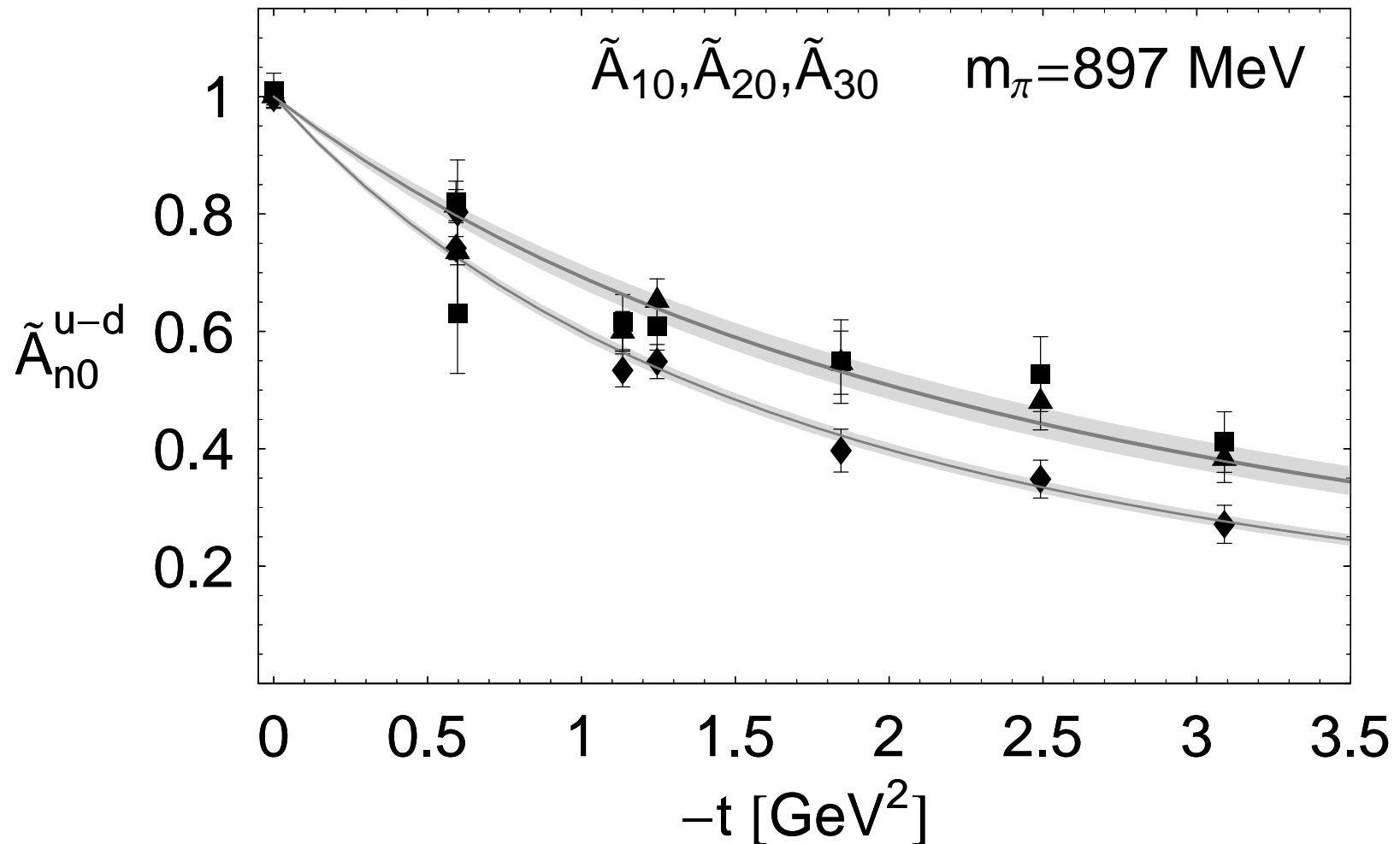
## Transverse Structure: Flavor Dependence

- slope of  $A_{10}^{u+d} = -1.38 \pm 0.03 \text{ (GeV)}^{-2}$
- slope of  $A_{30}^{u+d} = -0.45 \pm 0.07 \text{ (GeV)}^{-2}$  (factor of 3)



## Transverse Structure: Spin Dependence

- slope of  $\tilde{A}_{10}^{u-d} = -0.58 \pm 0.02 \text{ (GeV)}^{-2}$
- slope of  $\tilde{A}_{30}^{u-d} = -0.40 \pm 0.03 \text{ (GeV)}^{-2}$  (factor of 1.5)



## Transverse Structure

- transverse rms radius

$$\langle b_{\perp}^2 \rangle_x = \frac{\int d^2 b_{\perp} b_{\perp}^2 q(x, \vec{b}_{\perp})}{\int d^2 b_{\perp} q(x, \vec{b}_{\perp})}$$

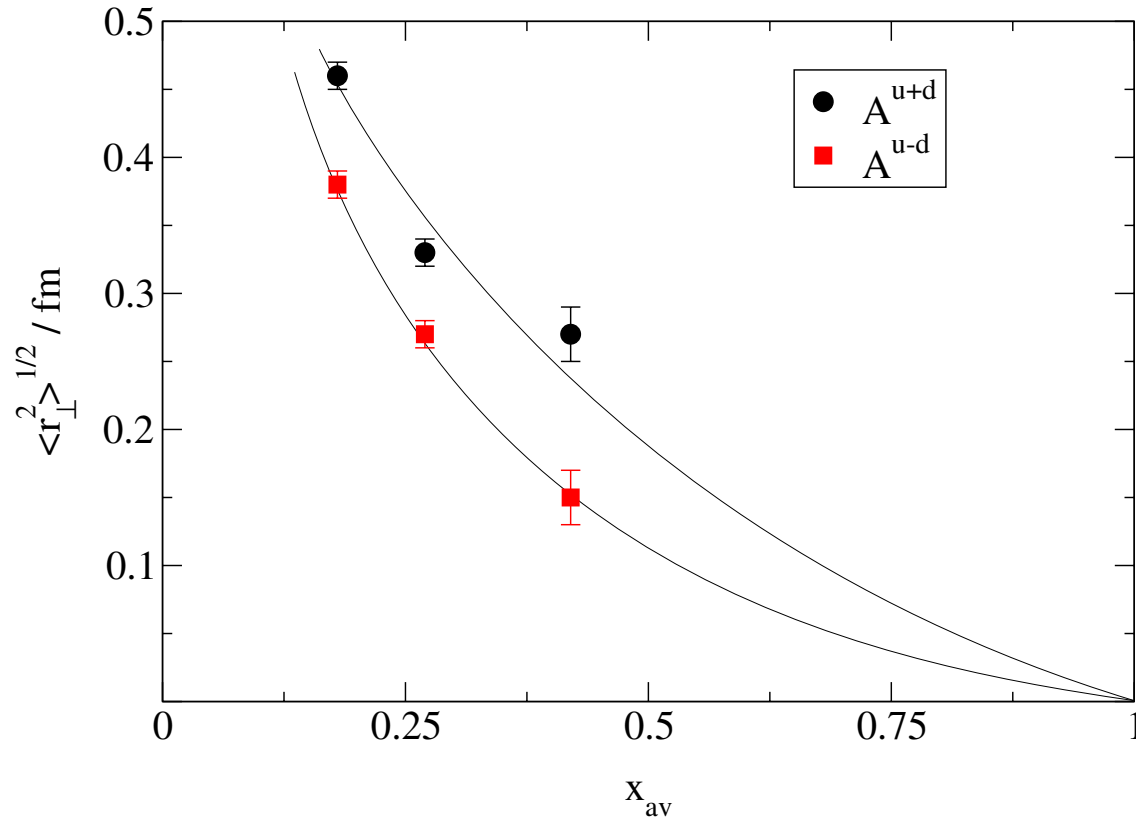
- smearred transverse rms radius

$$\frac{A'_{n0}(0)}{A_{n0}(0)} = -\frac{1}{4} \langle b_{\perp}^2 \rangle_{(n)} \quad \langle b_{\perp}^2 \rangle_{(n)} = \frac{\int d^2 b_{\perp} b_{\perp}^2 \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_{\perp})}{\int d^2 b_{\perp} \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_{\perp})}$$

- the average  $x$  in  $\langle b_{\perp}^2 \rangle_{(n)}$

$$x_{\text{av}}^{(n)} = \frac{\int d^2 b_{\perp} \int_{-1}^1 dx x \cdot x^{n-1} q(x, \vec{b}_{\perp})}{\int d^2 b_{\perp} \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_{\perp})} = \frac{\langle x^n \rangle}{\langle x^{n-1} \rangle}$$

# Smeared Transverse Structure



- charge radius:  $\sqrt{\langle b_{\perp}^2 \rangle}_{\text{charge}} = 0.42 \text{ fm}$  (0.72 fm from experiment), no pion cloud
- non-singlet radius:  $\sqrt{\langle b_{\perp}^2 \rangle_{u-d}^{(1)}} = 0.38 \text{ fm}$  &  $\sqrt{\langle b_{\perp}^2 \rangle_{u-d}^{(3)}} = 0.15 \text{ fm}$ , 61% decrease
- singlet radius:  $\sqrt{\langle b_{\perp}^2 \rangle_{u+d}^{(1)}} = 0.46 \text{ fm}$  &  $\sqrt{\langle b_{\perp}^2 \rangle_{u+d}^{(3)}} = 0.27 \text{ fm}$ , 41% decrease

## Current Work: Chiral Limit

- hybrid calculation: Asqtad staggered sea quarks (MILC) & domain wall valence quarks with HYP smearing
- $a = 0.13$  fm,  $L = 2.6$  fm,  $20^3 \times 32$
- $M_\pi = 343, 635$  MeV
- $O(100)$  gluon configurations
- extended statistics, intermediate masses, volume dependence

## Summary

- generalized parton distributions:
  - (1) contain form factors and ordinary parton distributions
  - (2) determine the spin decomposition of the nucleon
  - (3) measure Fourier transform of transverse quark distribution
- building blocks method to determine all matrix elements for parton distributions, form factors, and generalized parton distributions
- overdetermined observables method to accurately measure generalized form factors upto  $3 \text{ GeV}^2$
- in heavy pion world:  $\frac{1}{2} = \frac{1}{2}(0.68 + 0.0 + 0.32)$  & observed scaling in  $\frac{F_2}{F_1}$
- observed significant dependence of slopes of generalized form factors  $A_{n0}$  on  $n$  indicating a significant variation of transverse size with  $x$