Thesis Defense

Exploring Proton Structure Using Lattice QCD

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http://talks.drubryantrenner.org/thesis-defense.pdf

Generalized Parton Distributions from Lattice QCD

What new physics do we learn?

- Spin Decomposition - decomposition of nucleon spin into quark helicity, quark orbital, and gluon contributions

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma_{u+d} + L_{u+d} + J_g$$

- Transverse Structure - 3D distribution of quarks in a mixed representation: 2 transverse coordinates $\vec{b}_\perp$ and 1 longitudinal momentum x

$$q(x, \vec{b}_\perp) \quad \text{and} \quad \Delta q(x, \vec{b}_\perp)$$

What matrix elements determine the quark angular momenta
and transverse quark distributions?

Generalized Form Factors

- unpolarized and polarized twist two operators

$$O_q^{\mu_1\cdots\mu_n} = \overline{q} iD^{(\mu_1} \ldots iD^{\mu_{n-1}} \gamma^{\mu_n)} q$$
$$\tilde{O}_q^{\mu_1\cdots\mu_n} = \overline{q} iD^{(\mu_1} \ldots iD^{\mu_{n-1}} \gamma^{\mu_n)} \gamma^5 q$$

- off-forward matrix elements of the twist two operators

$$\langle P', S' | O_q^{\mu_1\cdots\mu_n} | P, S\rangle = \overline{U}(P', S')[\sum_{\substack{i=0 \\ \text{even}}}^{n-1} A_{ni}^q(t) K_{ni}^A(P', P)$$
$$+ \sum_{\substack{i=0 \\ \text{even}}}^{n-1} B_{ni}^q(t) K_{ni}^B(P', P) + \delta_{\text{even}}^n C_n^q(t) K_n^C(P', P)] U(P, S)$$

$$\langle P', S' | \tilde{O}_q^{\mu_1\cdots\mu_n} | P, S\rangle = \overline{U}(P', S')[\sum_{\substack{i=0 \\ \text{even}}}^{n-1} \tilde{A}_{ni}^q(t) \tilde{K}_{ni}^A(P', P) +$$
$$\sum_{\substack{i=0 \\ \text{even}}}^{n-1} \tilde{B}_{ni}^q(t) \tilde{K}_{ni}^B(P', P)] U(P, S)$$

Basic Properties of Generalized Form Factors

- moments of parton distributions

$$A^q_{n0}(0) = \int_{-1}^{1} dx\ x^{n-1}\, q(x) \quad \text{and} \quad \tilde{A}^q_{n0}(0) = \int_{-1}^{1} dx\ x^{n-1}\, \Delta q(x)$$

- form factors - $\mathcal{O}^\mu_q = \overline{q}\gamma^\mu q$ and $\tilde{\mathcal{O}}^\mu_q = \overline{q}\gamma^\mu\gamma^5 q$

$$A^q_{10}(t) = F^q_1(t) \quad \text{and} \quad B^q_{10}(t) = F^q_2(t)$$

$$\tilde{A}^q_{10}(t) = G^q_A(t) \quad \text{and} \quad \tilde{B}^q_{10}(t) = G^q_P(t)$$

Quark Angular Momenta and Transverse Quark Distributions

- quark angular momenta [1]

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

$$\Delta\Sigma^q = \tilde{A}^q_{10}(0) \qquad J^q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right) \qquad L^q = J^q - \frac{1}{2}\Delta\Sigma^q$$

- transverse quark distributions [2]

$$\int_{-1}^{1} dx\; x^{n-1}\, q(x, \vec{b}_\perp) \;=\; \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A^q_{n0}(-\vec{\Delta}^2_\perp)$$

$$\int_{-1}^{1} dx\; x^{n-1}\, \Delta q(x, \vec{b}_\perp) \;=\; \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \tilde{A}^q_{n0}(-\vec{\Delta}^2_\perp)$$

[1] X. D. Ji hep-ph/9603249
[2] M. Burkardt hep-ph/0005108

How do we calculate the matrix elements of twist two operators in lattice QCD?

Lattice Fields and Correlation Functions

- Grassmann fermion fields, $\psi_\alpha^a(x)$ and $\overline{\psi}_\alpha^a(x)$, at every lattice point x

- SU(3) gauge fields, $U_\mu^{ab}(x)$, at every lattice link $x \to x + \mu$

- correlation function of $2N$ fermion fields and M gauge links

$$\left\langle \psi_1 \cdots \psi_N \overline{\psi}_N \cdots \overline{\psi}_1 U_1 \cdots U_M \right\rangle =$$

$$= \int DU \int D\psi D\overline{\psi}\ e^{-S_G[U]} e^{-\overline{\psi} M[U] \psi} \psi_1 \cdots \psi_N \overline{\psi}_N \cdots \overline{\psi}_1 U_1 \cdots U_M$$

$$= \int DU\ e^{-S_G[U]}\ \mathrm{det}^{N_f}(M[U])\ U_1 \cdots U_M\ \sum_\pi (-1)^\pi M[U]^{-1}_{1\pi_1} \cdots M[U]^{-1}_{N\pi_N}$$

- numerical integration

$$\int DU\ e^{-S_G[U]}\ \mathrm{det}^{N_f}(M[U])\ F[U] = \frac{1}{N} \sum_{i=1}^{N} F[U^i] \pm \frac{\sigma_F}{\sqrt{N}}$$

Nucleon Fields on the Lattice

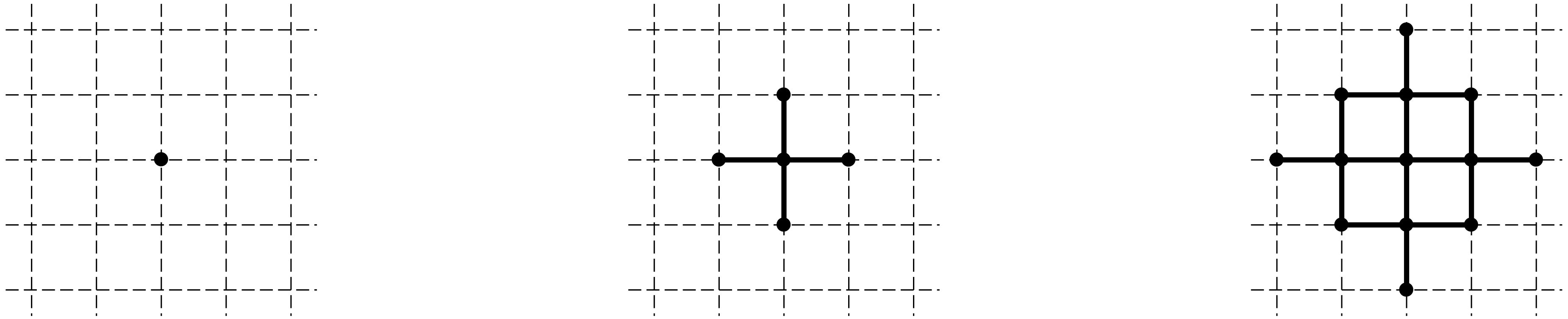
- nucleon interpolating field

$$P^{\alpha}(x) = U_a^{\alpha}(x)(U_b^T(x)C\gamma^5 D_c(x))\epsilon^{abc}$$

- gauge invariant extended or smeared quarks

$$Q^{(n)}(x) = \alpha Q^{(n-1)}(x) + \beta \sum_{\mu=\pm 1}^{\pm 4} U_{\mu}^{\dagger}(x-\mu)Q^{(n-1)}(x-\mu)$$

- iterative method



Nucleon Two Point Correlation Functions

- two point function

$$\langle P(x)\overline{P}(x')\rangle = \langle U(x)U(x)C\gamma^5 D(x)\overline{D}(x')\overline{C\gamma^5}\overline{U}(x')\overline{U}(x')\rangle$$

- quark diagrams

- spectral representation and nucleon mass, n labels all intermediate states in the nucleon channel

$$\begin{aligned}\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\langle P(t,\vec{x})\overline{P}(t',\vec{0})\rangle &= \sum_n \langle\Omega|P|n,\vec{p}\rangle e^{-E^n_{\vec{p}}(t-t')}\langle n,\vec{p}|\overline{P}|\Omega\rangle \\ &\rightarrow \langle\Omega|P|\vec{p}\rangle e^{-E_{\vec{p}}(t-t')}\langle\vec{p}|\overline{P}|\Omega\rangle\end{aligned}$$

Nucleon Three Point Correlation Functions

- three point function

$$\langle P(x)\overline{q}(y')q(y)\overline{P}(x')\rangle = \langle U(x)U(x)C\gamma^5 D(x)\overline{q}(y')q(y)\overline{D}(x')\overline{C\gamma^5}\,\overline{U}(x')\overline{U}(x')\rangle$$

- 10 quark diagrams - next slide

- spectral representation and nucleon matrix elements, n and m label all intermediate states in the nucleon channel

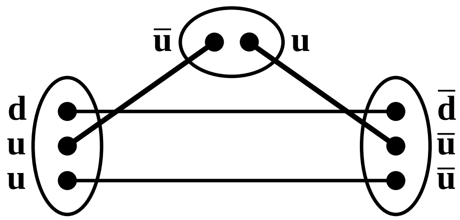
$$\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \sum_{\vec{y}} e^{i(\vec{p}-\vec{k})\cdot\vec{y}} \langle P(t,\vec{x})O(\tau,\vec{y})\overline{P}(t',\vec{0})\rangle =$$

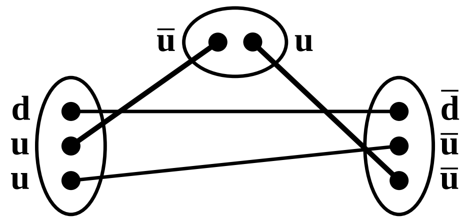
$$= \sum_{n\,m}\sum_{s\,r} \langle\Omega|P|n,\vec{p},s\rangle e^{-E^n_{\vec{p}}(t-\tau)} \langle n,\vec{p},s|O|m,\vec{k},r\rangle e^{-E^m_{\vec{k}}(\tau-t')} \langle m,\vec{k},r|\overline{P}|\Omega\rangle$$

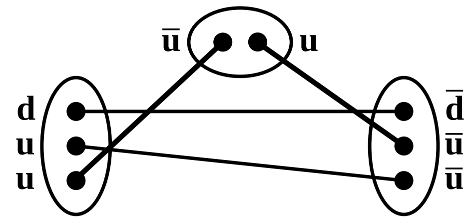
$$\to \sum_{s\,r} \langle\Omega|P|\vec{p},s\rangle e^{-E_{\vec{p}}(t-\tau)} \langle\vec{p},s|O|\vec{k},r\rangle e^{-E_{\vec{k}}(\tau-t')} \langle\vec{k},r|\overline{P}|\Omega\rangle$$

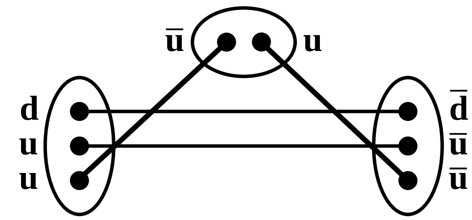
Quark Diagrams

- connected u quark diagrams

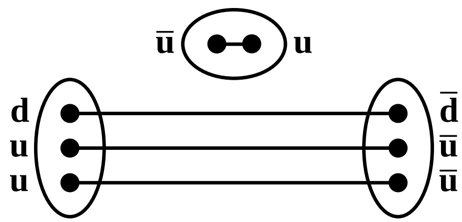


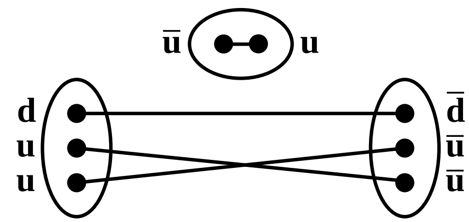




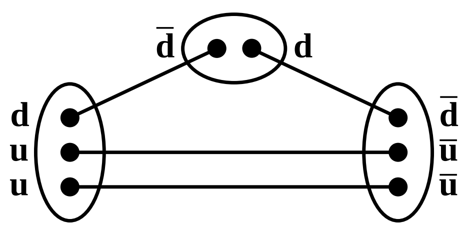


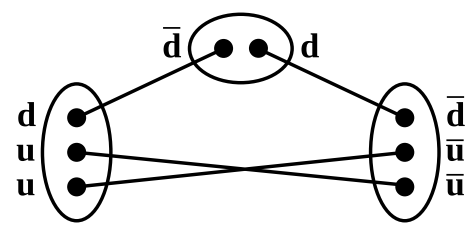
- disconnected u diagrams - not calculated - $(u + d)$ matrix elements only



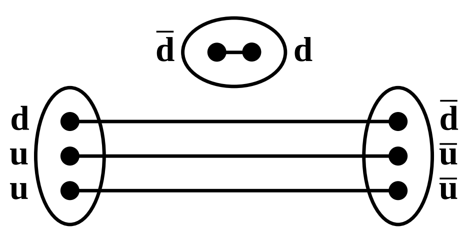


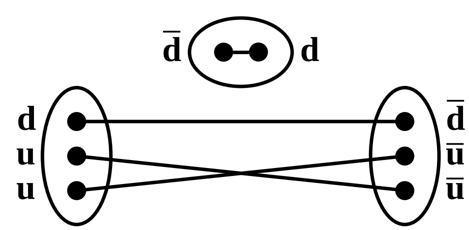
- connected d quark diagrams





- disconnected d diagrams - not calculated - $(u + d)$ matrix elements only

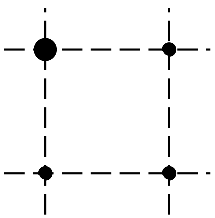


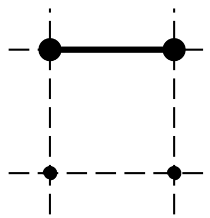


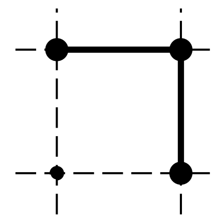
Building Blocks

- twist two operators, $\overline{q}\Gamma D^{\mu_1}\cdots D^{\mu_n}q$, can be written in terms of a basic set of building blocks

$\overline{q}(x)\,\Gamma_i\,q(x)$ $\quad$ $\overline{q}(x+\widehat{\mu})\,\Gamma_i\,U^{\dagger}_{\mu}(x)\,q(x)$ $\quad$ $\overline{q}(x+\widehat{\nu}+\widehat{\mu})\,\Gamma_i\,U^{\dagger}_{\nu}(x+\widehat{\mu})U^{\dagger}_{\mu}(x)\,q(x)$







- Γ_i $(i=1,\ldots,16)$ denote a complete basis of 4×4 Dirac spin matrices

- calculate upto 3 gauge link insertions: $U^{\dagger}_{\mu_n}(x+\widehat{\mu}_{n-1}+\cdots\widehat{\mu}_1)\cdots U^{\dagger}_{\mu_1}(x)$

- calculate a broad range of momentum transfers $\vec{q}$

- more than $380,000$ correlation functions

Overdetermined Set of Lattice Observables

- i labels all combinations of operator indices $(\mu_1, \ldots, \mu_n)$ and momenta P', P which give the same $t = (P' - P)^2$

$$\mathcal{O}_i^{\overline{\mathsf{MS}}} = \left\langle P' | \mathcal{O}^{\mu_1 \cdots \mu_n} | P \right\rangle$$

- expand $\mathcal{O}_i^{\overline{\mathsf{MS}}}$ in terms of generalized form factors, $F_\alpha = A_{ni}$, B_{ni}, $\ldots$

$$\mathcal{O}_i^{\overline{\mathsf{MS}}} = \sum_{\alpha=1}^{M} K_{i\alpha} F_\alpha(t)$$

- match $\mathcal{O}_i^{\overline{\mathsf{MS}}}$ to $\mathcal{O}_i^{\mathsf{lat}}$ with one loop matching coefficients Z_{ij}

$$\mathcal{O}_i^{\overline{\mathsf{MS}}} = \sum_{j=1}^{N} Z_{ij} \mathcal{O}_j^{\mathsf{lat}}$$

- overdetermined set of observables, $N > M$

$$\mathcal{O}_i^{\mathsf{lat}} = \sum_\alpha (Z^{-1}K)_{i\alpha} F_\alpha(t) \qquad \chi^2 = \sum_{i=1}^{N} \left(\frac{\sum_{\alpha=1}^{M} (Z^{-1}K)_{i\alpha} F_\alpha(t) - \mathcal{O}_i^{\mathsf{lat}}}{\sigma_i} \right)^2$$

What gauge coupling and quark masses did we use?

Lattice Action, Gauge Coupling, Quark and Pion Masses

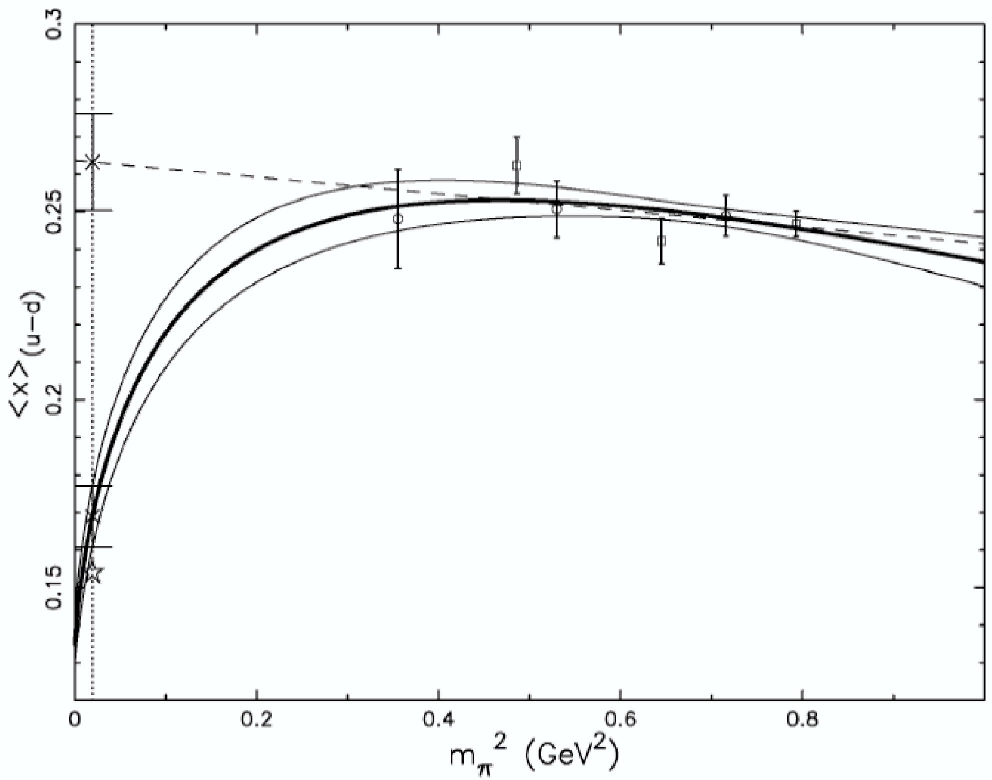
- SESAM gauge fields
- Wilson gluons and $N_F = 2$ Wilson fermions
- $a = 0.095$ fm
- $m_q^{\overline{\text{MS}}}(2\ \text{GeV}) = 79,\ 101,\ 123$ MeV
- $M_\pi = 753(10),\ 835(13),\ 895(15)$ MeV

Chiral Extrapolations or Heavy Pion World?

- leading order χPT, nucleon core of size $1/\mu$, heavy quark limit

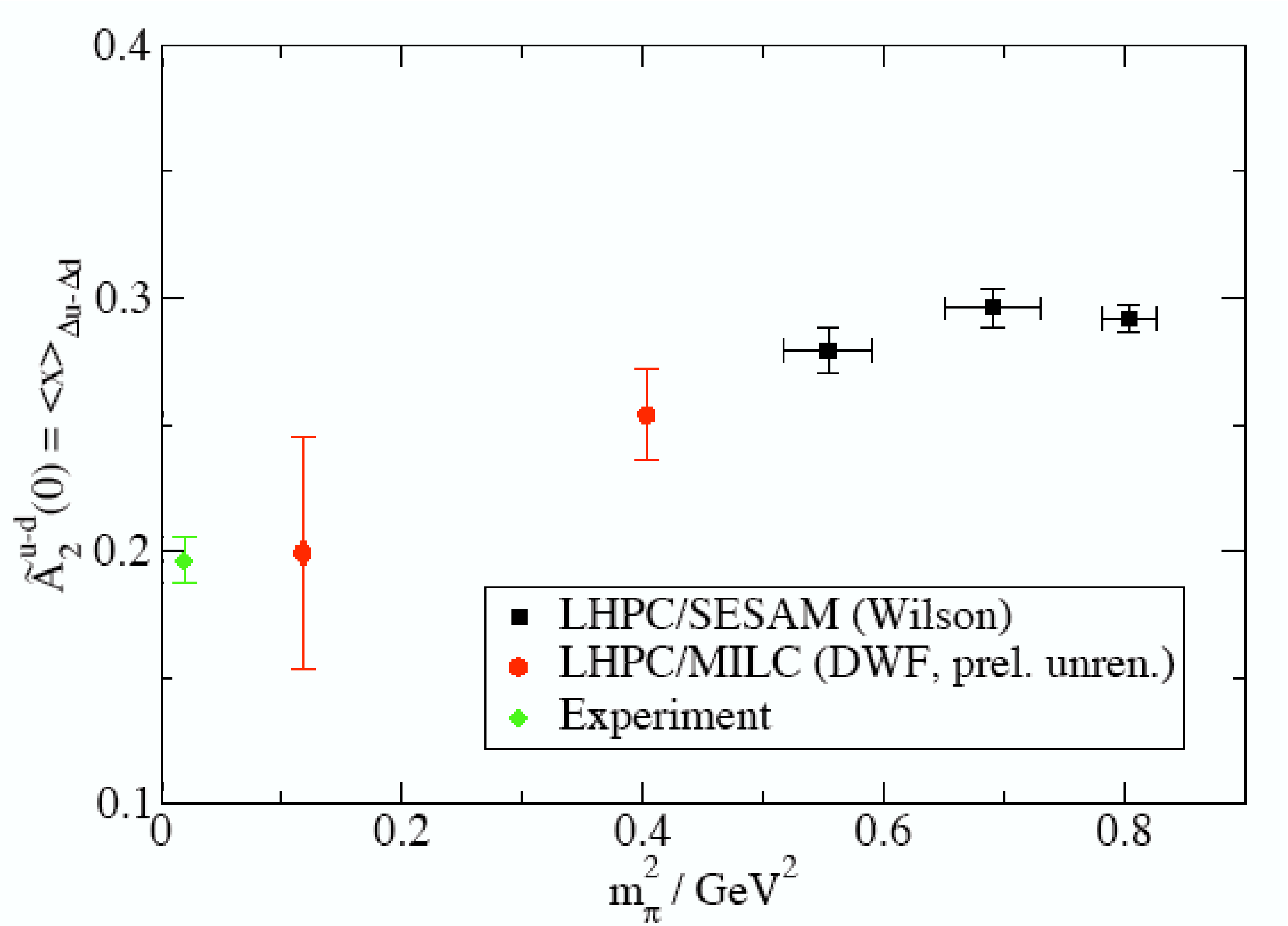
$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_A^2+1)m_\pi^2}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)\right) + b_n m_\pi^2$$

- $\langle x \rangle$ for SESAM full QCD, MIT quenched



Chiral Explorations

- $\langle x \rangle_{\Delta u - \Delta d}$ for staggered asqtad sea quarks and domain wall valence quarks



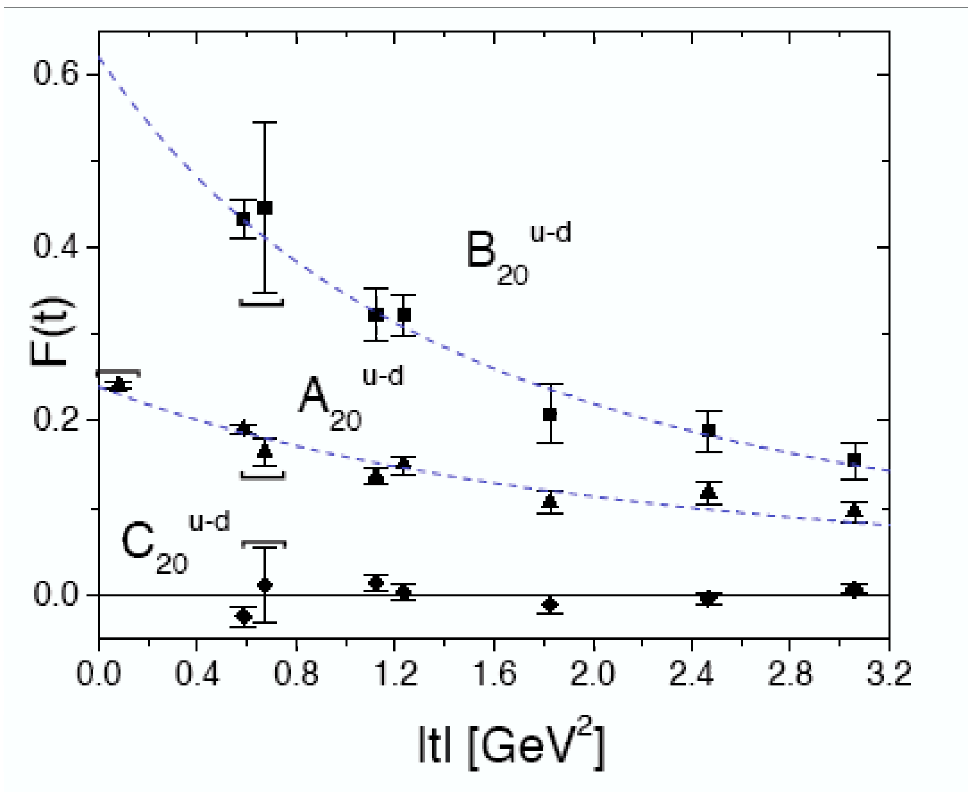
What do we learn about the nature of the nucleon spin?

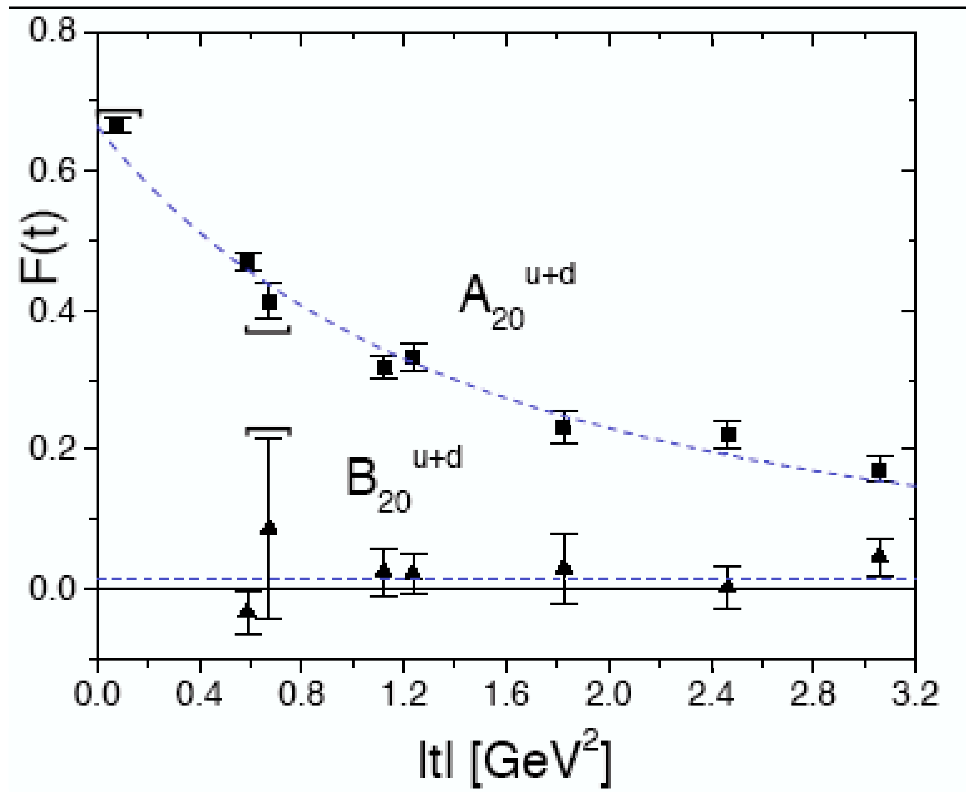
Quark Angular Momenta

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

$$\Delta\Sigma_q = \frac{1}{2}\tilde{A}^q_{10}(0) \qquad L_q = J_q - \frac{1}{2}\Delta\Sigma_q$$

$$J_q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right) \qquad J_g = \frac{1}{2} - J_{u+d}$$





Quark Angular Momenta: $m_\pi = 895$ MeV

- flavor structure

$$2 \cdot \frac{1}{2} = \begin{array}{ccc} 2J^u & +2J^d & +2J^g \\ +0.742(31) & -0.062(31) & +0.320(16) \end{array}$$

- helicity, orbital, gluon contributions

$$2 \cdot \frac{1}{2} = \begin{array}{ccc} \Delta\Sigma^{u+d} & +2L^{u+d} & +2J^g \\ +0.682(20) & -0.002(3) & +0.320(16) \end{array}$$

- complete decomposition

$$2 \cdot \frac{1}{2} = \begin{array}{ccccc} \Delta\Sigma^u & +\Delta\Sigma^d & +2L^u & +2L^d & +2J^g \\ +0.936(14) & -0.254(14) & -0.194(34) & +0.192(34) & +0.320(16) \end{array}$$

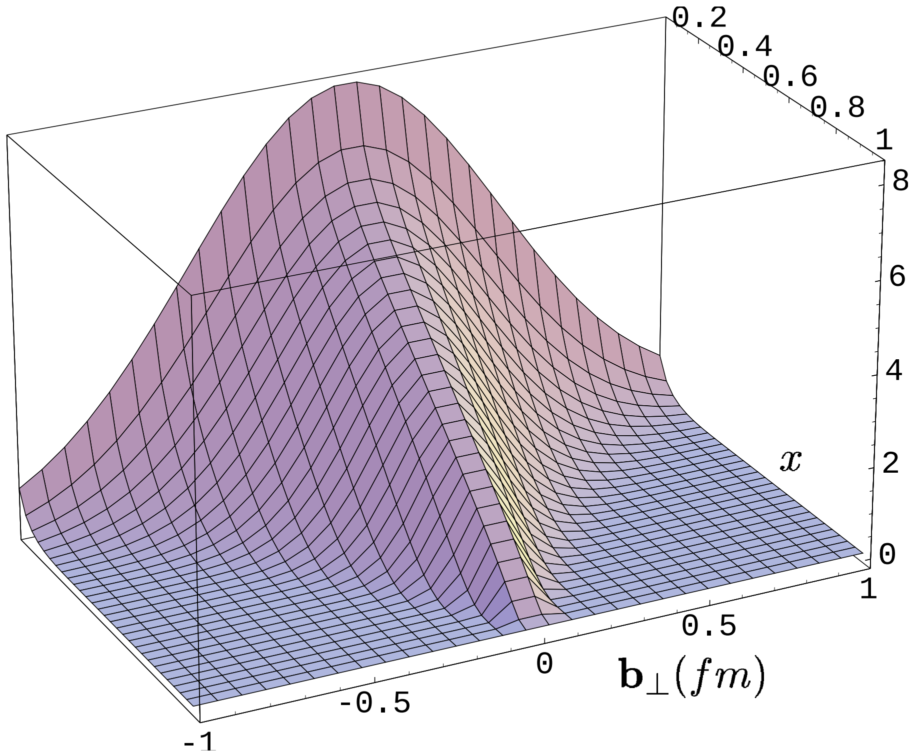
- quark orbital motion, $L^{u-d} = -0.193(32)$

$$L^q_{\mathrm{min}} = 2 \cdot \left(\frac{|L^u| + |L^d|}{2} \right) \geq |L^{u-d}| = 0.193(32)$$

What do we learn about the transverse quark structure of the nucleon?

Transverse Structure

- 3D quark distribution: $q(x, \vec{b}_\perp)$
- x is longitudinal momentum fraction
- $\vec{b}_\perp$ is transverse displacement of active parton relative to nucleon center of longitudinal momentum
- simple model calculation



Burkardt hep-ph/0207047

Transverse Structure

$$A^q_{n0}(-\vec{\Delta}^2_\perp) = \int d^2b_\perp \, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} \, q(x,\vec{b}_\perp)$$

$$\langle b^2_\perp \rangle^q_{(n)} = -4 \frac{A^{q\,\prime}_{n0}(0)}{A^q_{n0}(0)}$$

- at $x = 1$ a single quark carries all the momentum

$$\lim_{x\to 1} q(x,\vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

- higher moments A^q_{n0} weight $x \sim 1$ more heavily

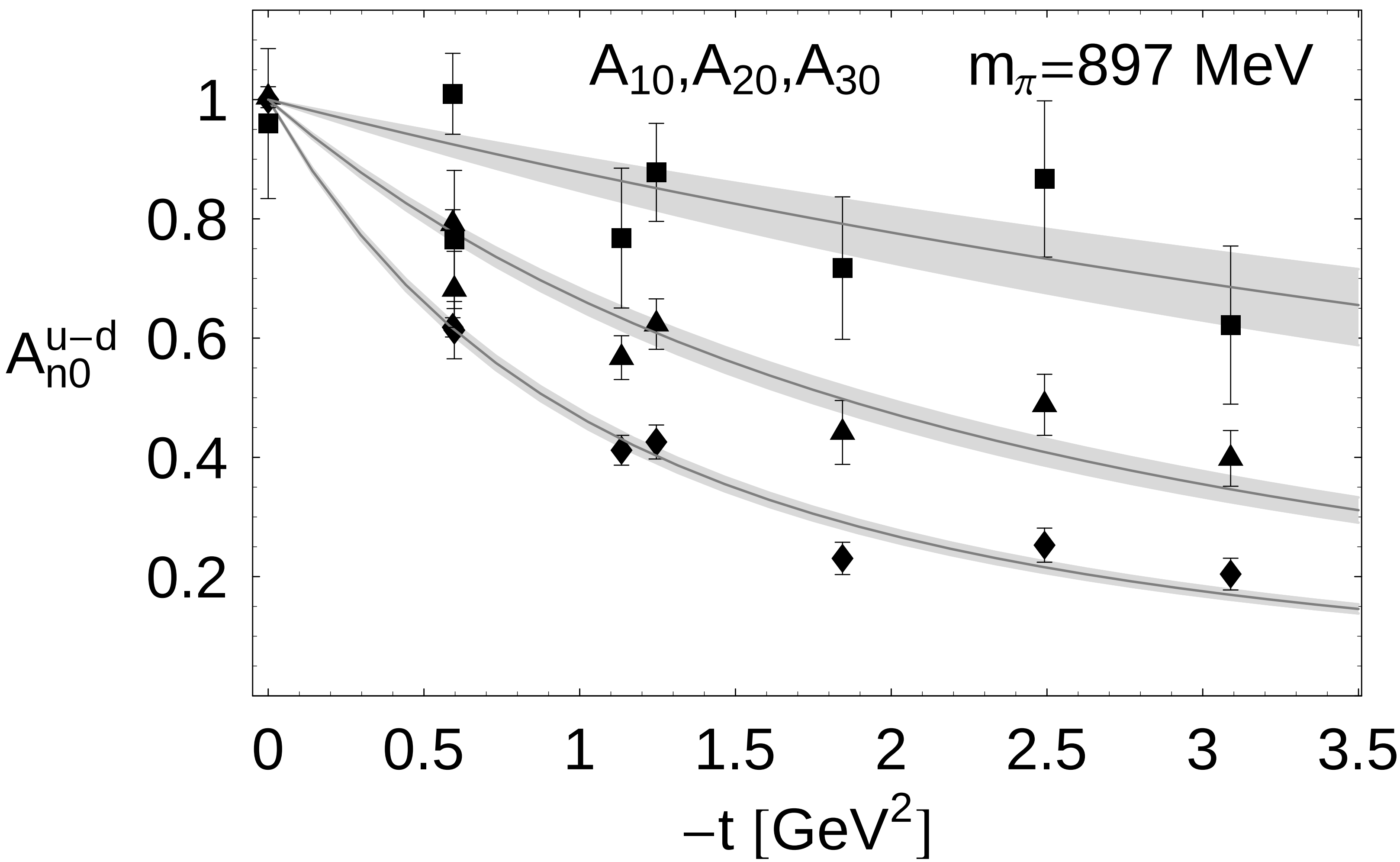
$$\lim_{n\to\infty} A^q_{n0}(t) \propto \int d^2b_\perp \, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \delta^2(\vec{b}_\perp) = \text{constant}$$

- slopes of A^q_{n0} should decrease as n increases

- A_{10}, A_{30}, $\tilde{A}_{20}$ measure $q - \bar{q}$ & $\tilde{A}_{10}$, $\tilde{A}_{30}$, A_{20} measure $q + \bar{q}$

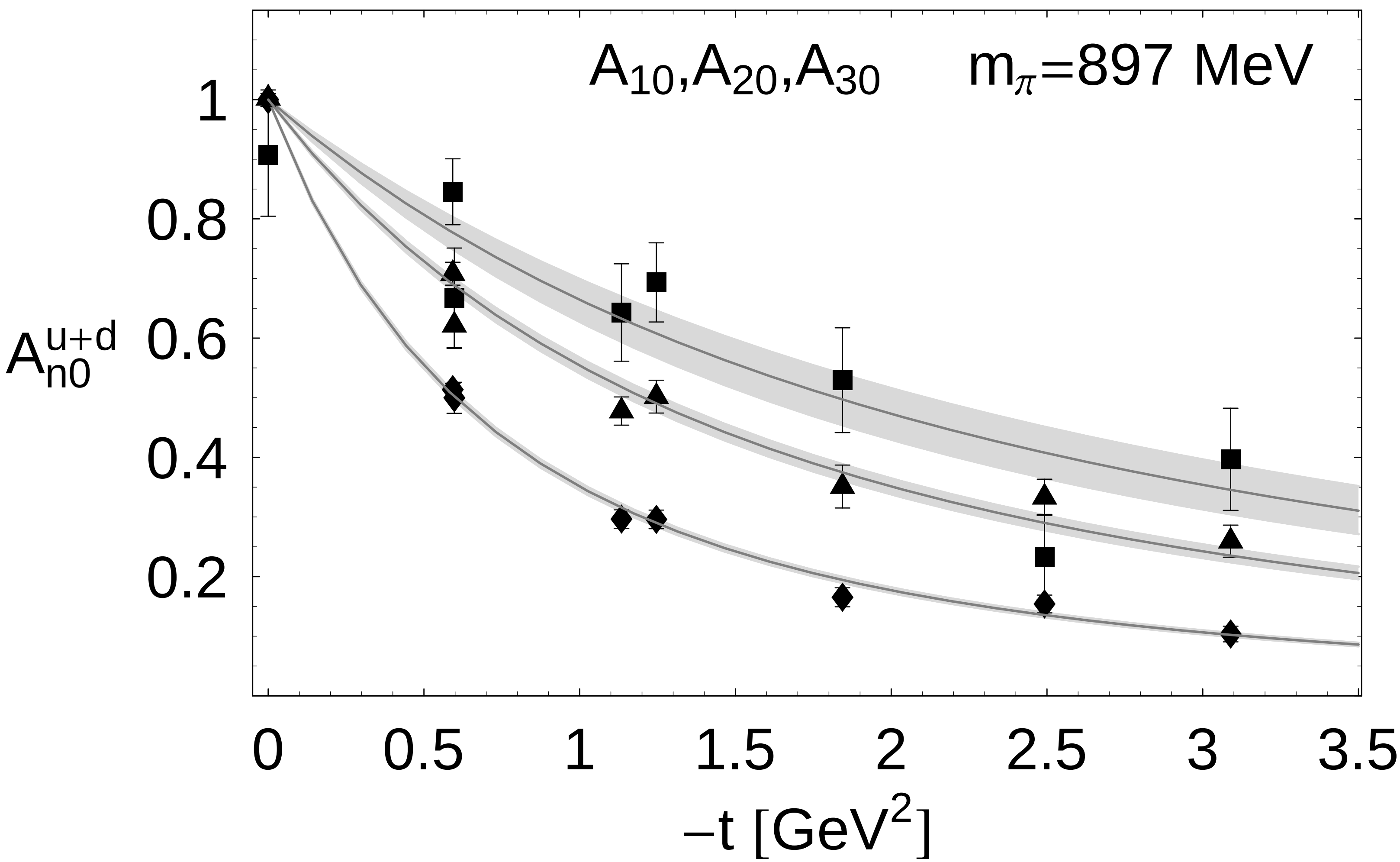
Transverse Distributions: $m_\pi = 897$ MeV

- slope of $A_{10}^{u-d} = -0.93 \pm 0.04$ (GeV)$^{-2}$
- slope of $A_{30}^{u-d} = -0.13 \pm 0.03$ (GeV)$^{-2}$ (factor of 7)



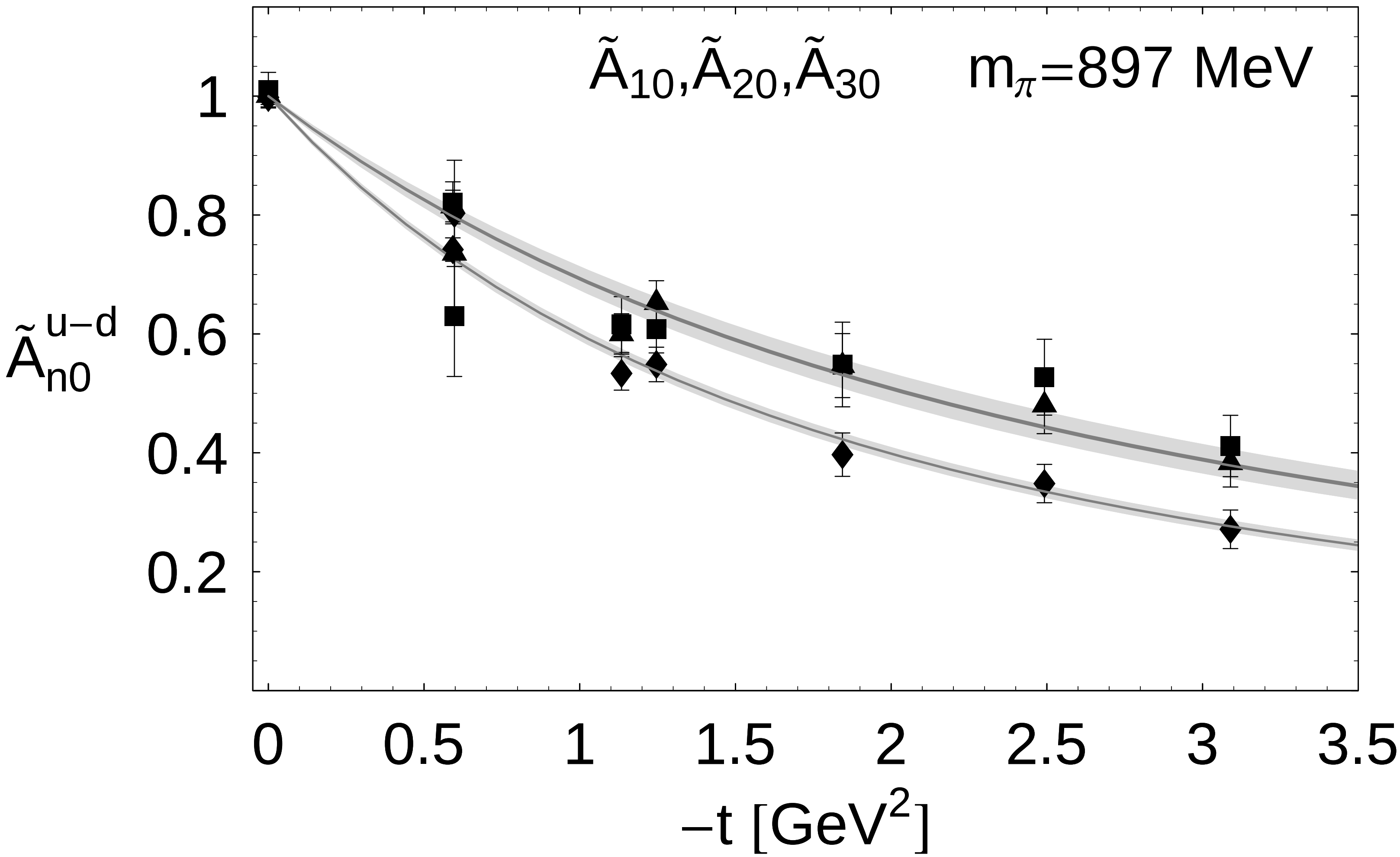
Transverse Distributions: Flavor Dependence

- slope of $A_{10}^{u+d} = -1.38 \pm 0.03\ (\text{GeV})^{-2}$
- slope of $A_{30}^{u+d} = -0.45 \pm 0.07\ (\text{GeV})^{-2}$ (factor of 3)



Transverse Distributions: Spin Dependence

- slope of $\tilde{A}_{10}^{u-d} = -0.58 \pm 0.02\ (\mathrm{GeV})^{-2}$
- slope of $\tilde{A}_{30}^{u-d} = -0.40 \pm 0.03\ (\mathrm{GeV})^{-2}$ (factor of 1.5)



Transverse Distributions: x Behavior

- transverse rms radius

$$\left\langle b_\perp^2 \right\rangle_x = \frac{\int d^2 b_\perp \, b_\perp^2 \, q(x,\vec{b}_\perp)}{\int d^2 b_\perp \, q(x,\vec{b}_\perp)}$$

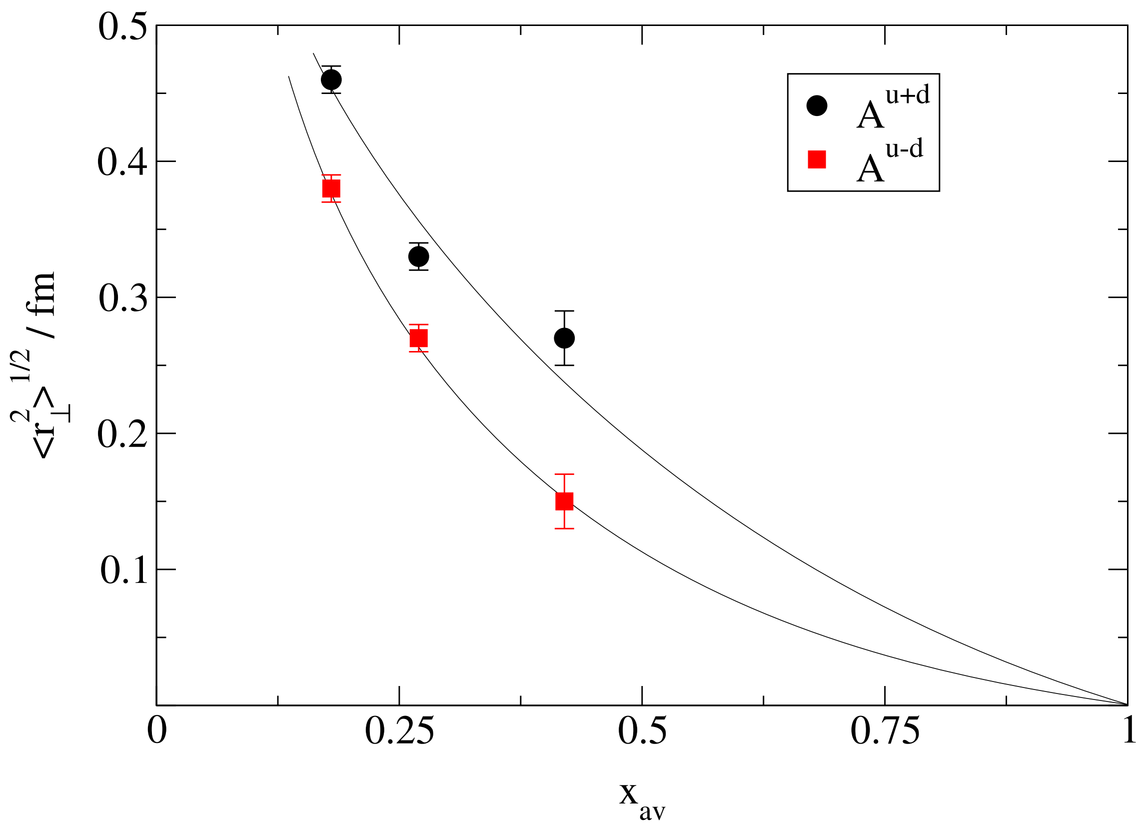
- transverse rms *moment* radius

$$\frac{A'_{n0}(0)}{A_{n0}(0)} = -\frac{1}{4}\left\langle b_\perp^2 \right\rangle_{(n)} \qquad \left\langle b_\perp^2 \right\rangle_{(n)} = \frac{\int d^2 b_\perp \, b_\perp^2 \, \int_{-1}^{1} dx \, x^{n-1} q(x,\vec{b}_\perp)}{\int d^2 b_\perp \, \int_{-1}^{1} dx \, x^{n-1} q(x,\vec{b}_\perp)}$$

- the average x in $\left\langle b_\perp^2 \right\rangle_{(n)}$

$$x_{\text{av}}^{(n)} = \frac{\int d^2 b_\perp \, \int_{-1}^{1} dx \, x \cdot x^{n-1} q(x,\vec{b}_\perp)}{\int d^2 b_\perp \, \int_{-1}^{1} dx \, x^{n-1} q(x,\vec{b}_\perp)} = \frac{< x^n >}{< x^{n-1} >}$$

Transverse Distributions: x Behavior

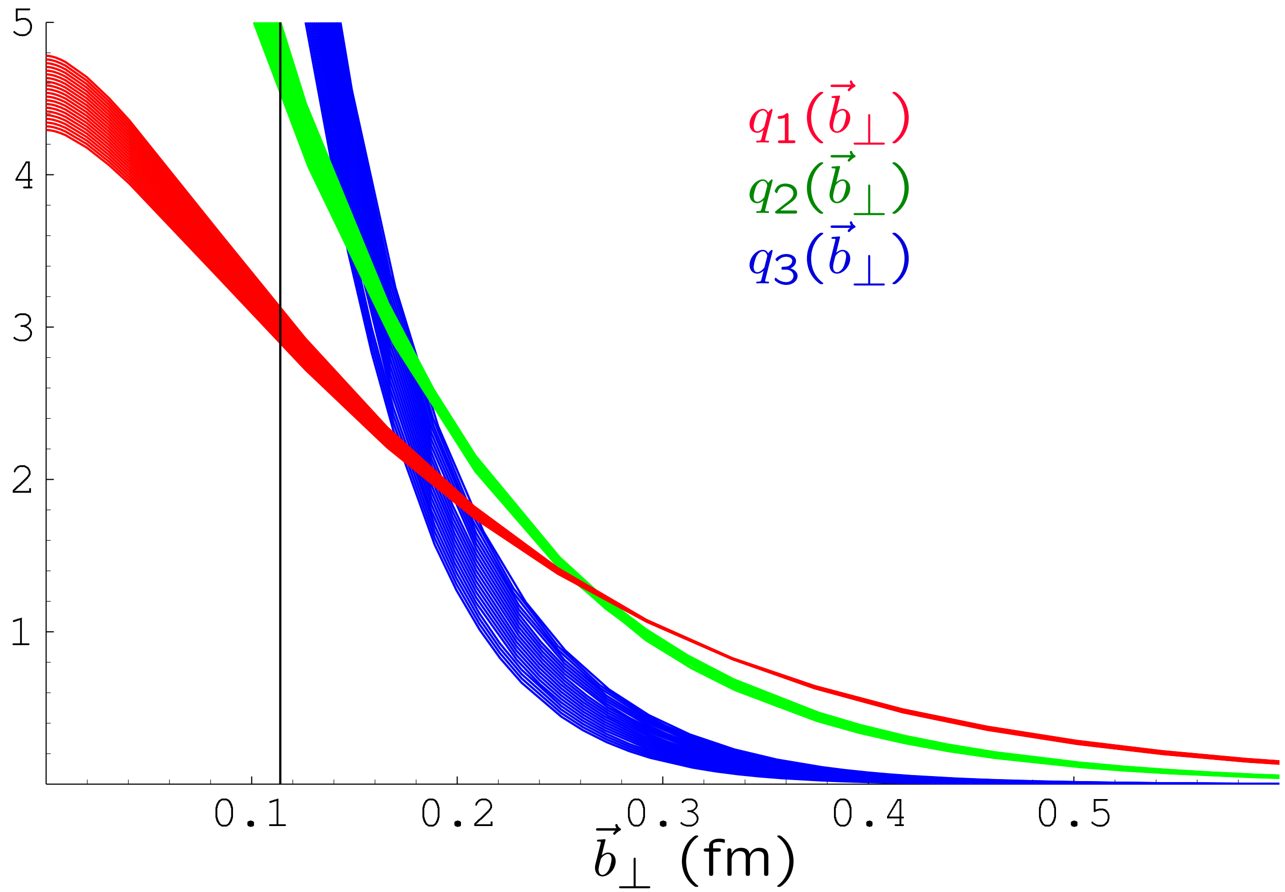


- non-singlet radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{u-d}^{(1)}} = 0.38$ fm & $\sqrt{\left\langle b_\perp^2 \right\rangle_{u-d}^{(3)}} = 0.15$ fm, 61% decrease
- singlet radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{u+d}^{(1)}} = 0.46$ fm & $\sqrt{\left\langle b_\perp^2 \right\rangle_{u+d}^{(3)}} = 0.27$ fm, 41% decrease

Transverse Distributions: $\vec{b}_\perp$ Behavior

$$q_1(\vec{b}_\perp) = \int_{-1}^{1} dx\; q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{10}^q(-\vec{\Delta}_\perp^2)$$

$$q_2(\vec{b}_\perp) = \int_{-1}^{1} dx\; x\, q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{20}^q(-\vec{\Delta}_\perp^2)$$



Summary

- building blocks method to determine all matrix elements for parton distributions, form factors, and generalized parton distributions

- overdetermined observables method to accurately measure generalized form factors upto and beyond 3 GeV^2

- physics results in a heavy pion world:

  (1) $\frac{1}{2} = \frac{1}{2}(0.68(2) - 0.002(3) + 0.32(2))$

  (2) $L_{u-d} = -0.19(3)$

  (3) calculated the transverse size as a function of x

  (4) examined the $\vec{b}_\perp$ dependence of moments of quark distributions

Publications

hep-lat/0404005
hep-lat/0312014
hep-lat/0309065
hep-lat/0309060
hep-lat/0304018
hep-lat/0211021
hep-lat/0211019
hep-lat/0201021
hep-lat/0103006
hep-lat/0011010

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