Generalized Parton Distributions

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MILC Meeting

July 14, 2004

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http://talks.drubryantrenner.org/milc_7-14-04.pdf

In Short

Generalized Parton Distributions determine

- 3D distribution of quarks in a mixed representation - 2 transverse coordinates and 1 longitudinal momentum

- decomposition of nucleon spin into quark helicity, quark orbital, and gluon contributions

- form factors and ordinary parton distributions

Continuum Physics

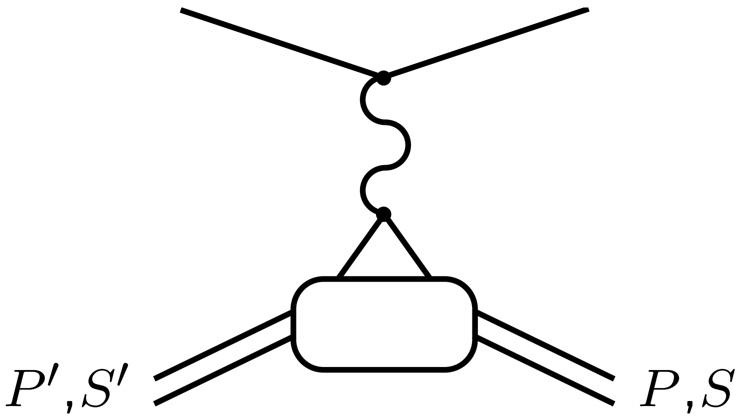
I. Form Factors

II. Parton Distributions

III. Generalized Parton Distributions

Form Factors

- lepton-nucleon scattering, $lN \to lN$, elastic



$$\Delta = P' - P$$
$$t = \Delta^2$$

- off-forward matrix element of the electromagnetic current

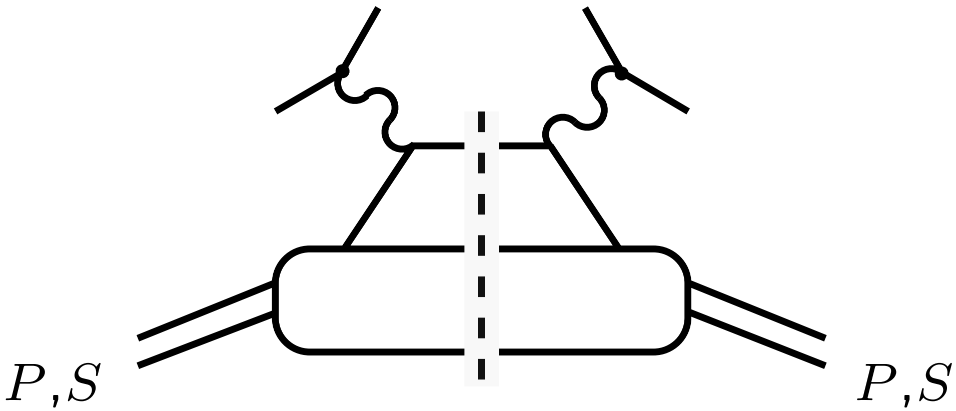
$$< P', S'|J^\mu|P, S >= \overline{U}(P', S')\left(\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}F_2(t)\right)U(P, S)$$

- interpretation as Fourier transform of charge and current densities *in certain cases*

- magnetic moment, charge & current radii

Parton Distributions

- deep inelastic scattering, $lN \to lX$, inclusive



$$P^+ = \frac{1}{2}\left(P^t + P^z\right)$$
$$y^- = \frac{1}{2}\left(y^t - y^z\right)$$

- forward matrix element of light-cone quark correlator

$$\mathcal{O}_q(x, \vec{b}_\perp) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \, \overline{q}\left(-\frac{y^-}{2}, \vec{b}_\perp\right) \gamma^+ q\left(\frac{y^-}{2}, \vec{b}_\perp\right)$$

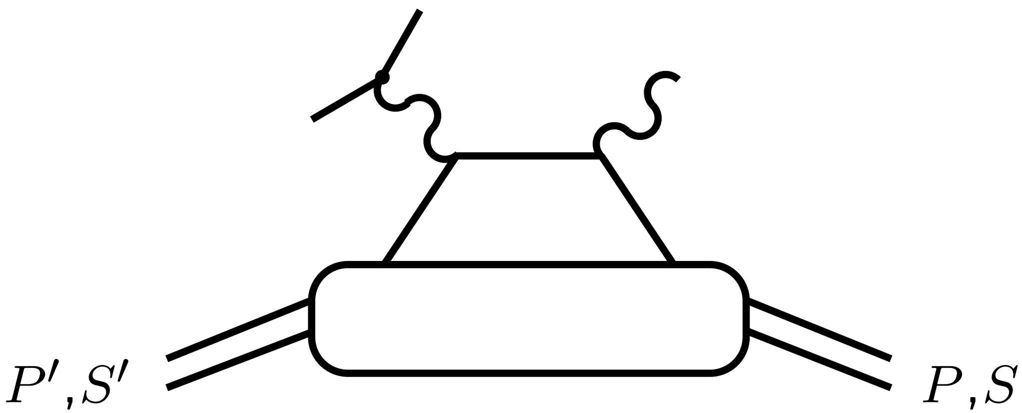
$$\left\langle P, S \middle| \mathcal{O}_q(x, \vec{0}_\perp) \middle| P, S \right\rangle = q(x)$$

- longitudinal momentum distribution in the *infinite momentum frame*

- momentum fraction & spin fraction

Generalized Parton Distributions

- deeply virtual Compton scattering, $lN \to lN\gamma$, exclusive



$$\overline{P} = \tfrac{1}{2}\left(P' + P\right)$$

$$\xi = -\frac{\Delta^+}{2\overline{P}^+}$$

- off-forward matrix element of light-cone quark correlator

$$\left\langle P', S' \middle| \mathcal{O}_q(x, \vec{0}_\perp) \middle| P, S \right\rangle =$$

$$\frac{1}{2\overline{P}^+}\overline{U}(P', S')\left(\gamma^+ H(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M}E(x,\xi,t)\right)U(P,S)$$

- meson production, Compton scatering

GPD reviews: Ji hep-ph/9807358, Diehl hep-ph/0307382

Generalized Parton Distributions

- familiar limits: parton distributions & form factors

$$H_q(x,0,0) = q(x)$$

$$\sum_q e_q \int dx\ H_q(x,\xi,t) = F_1(t)$$

$$\sum_q e_q \int dx\ E_q(x,\xi,t) = F_2(t)$$

- quark angular momentum [1]

$$J_q = \frac{1}{2}\int dx\ x\left(H_q(x,0,0) + E_q(x,0,0)\right)$$

- transverse quark distribution in *infinite momentum frame* [2]

$$\int d^2b_\perp\ e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} q(x,\vec{b}_\perp) = H_q(x,0,-\vec{\Delta}_\perp^2)$$

[1] Ji hep-ph/9603249
[2] Burkardt hep-ph/0005108

Transverse Structure

I. Charge Distribution: Non-Relativistic Limit

II. Transverse Distribution: Infinite Momentum Frame

Charge Distribution

- wave packet

$$|\psi\rangle = \int \frac{d^3p}{(2\pi)^{3/2}} \frac{\psi(\vec{p})}{\sqrt{2E_{\vec{p}}}} |\vec{p}\rangle$$

- Fourier transform of charge distribution

$$\begin{aligned}
&\int d^3x\, e^{i\vec{q}\cdot\vec{x}} \, \langle\psi|\, J^0(\vec{x})\, |\psi\rangle = \\
&= \int d^3p\, \psi^*(\vec{k})\, \psi(\vec{p}) \, <\vec{k}|J^0(0)|\vec{p}> / \left(2\sqrt{E_{\vec{k}}E_{\vec{p}}}\right) \\
&= \int d^3p\, \psi^*(\vec{k})\, \psi(\vec{p})\, F(q^2) \, \left(E_{\vec{k}} + E_{\vec{p}}\right) / \left(2\sqrt{E_{\vec{k}}E_{\vec{p}}}\right)
\end{aligned}$$

where $\vec{k} = \vec{p} + \vec{q}$ & $<\vec{k}|J^0(0)|\vec{p}> = F(q^2)\left(E_{\vec{k}} + E_{\vec{p}}\right)$

Burkardt hep-ph/0005108

Charge Distribution

- non-relativistic limit $\quad |\vec{q}| \ll M \quad E_{\vec{k}} \approx E_{\vec{p}} \quad q^2 \approx -\vec{q}^{\,2}$
- broad wave packet $\quad \psi(\vec{k}) \approx \psi(\vec{p})$

$$
\begin{aligned}
&\int d^3x \; e^{i\vec{q}\cdot\vec{x}} \, \langle\psi| \, J^0(\vec{x}) \, |\psi\rangle \\
&= \int d^3p \; \psi^*(\vec{k}) \; \psi(\vec{p}) \; F(q^2) \; \left(\frac{E_{\vec{k}} + E_{\vec{p}}}{2\sqrt{E_{\vec{k}} E_{\vec{p}}}} \right) \\
&\approx \int d^3p \; \psi^*(\vec{p}) \; \psi(\vec{p}) \; F(-\vec{q}^{\,2}) \\
&\approx F(-\vec{q}^{\,2})
\end{aligned}
$$

Charge Distribution

- separation of scales, $L_{FF/WP}$ is the form factor/wave packet length scale

$$L_{FF} \sim \frac{1}{|\vec{q}|} \gg L_{WP} \gg \frac{1}{M} \qquad \Rightarrow \qquad L_{FF} \gg \frac{1}{M}$$

- hydrogen atom

$$L_{FF} = \frac{1}{\alpha m_e} = 7 \cdot 10^4 \text{ fm} \quad \gg \quad \frac{1}{M_H} = \frac{1}{M_P} = 0.2 \text{ fm}$$

- proton

$$L_{FF} = 0.87 \text{ fm} \quad \approx \quad \frac{1}{M_P} = 0.2 \text{ fm}$$

Belitsky, Ji, Yuan hep-ph/0307383

Transverse Distribution

- wave packet

$$|\psi\rangle = \int \frac{d^2 p_\perp}{(2\pi)} \frac{\psi(\vec{p}_\perp)}{\sqrt{2E_{\vec{p}}}} |\vec{p}_\perp, P_z\rangle \qquad \vec{p} = (\vec{p}_\perp, P_z)$$

- Fourier transform of transverse quark distribution

$$\mathcal{O}_q(x, \vec{b}_\perp) = \int \frac{dy^-}{4\pi} e^{ix\overline{P}^+ y^-} \, \overline{q}\left(-y^-/2, \vec{b}_\perp\right) \gamma^+ q\left(y^-/2, \vec{b}_\perp\right)$$

$$\int d^2 b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \langle\psi| \mathcal{O}_q(x, \vec{b}_\perp) |\psi\rangle =$$

$$\int d^2 p_\perp \, \psi^*(\vec{k}_\perp) \, \psi(\vec{p}_\perp) \, H_q(x, 0, \Delta^2) \left(2\sqrt{E_{\vec{k}}}\sqrt{E_{\vec{p}}}\right)$$

$$\vec{k}_\perp = \vec{p}_\perp + \vec{q}_\perp \qquad k_z = p_z = P_z \qquad < \vec{k}|\mathcal{O}_q(x, \vec{0}_\perp)|\vec{p} >= H_q(x, 0, \Delta^2)$$

Transverse Distribution

- infinite momentum limit $\quad |\vec{q}_\perp| \ll P_z \quad E_{\vec{k}} \approx E_{\vec{p}} \quad \Delta^2 \approx -\vec{\Delta}_\perp^2$

- broad wave packet $\quad \psi(\vec{k}_\perp) \approx \psi(\vec{p}_\perp)$

$$
\begin{aligned}
&\int d^2 b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \langle \psi | \, \mathcal{O}_q(x, \vec{b}_\perp) \, | \psi \rangle \\
&= \int d^2 p_\perp \; \psi^*(\vec{k}_\perp) \; \psi(\vec{p}_\perp) \; H_q(x, 0, \Delta^2) \, / \left(2\sqrt{E_{\vec{k}}}\sqrt{E_{\vec{p}}} \right) \\
&\approx \int d^2 p_\perp \; \psi^*(\vec{p}_\perp) \; \psi(\vec{p}_\perp) \; H_q(x, 0, -\vec{\Delta}_\perp^2)/2P_z \\
&\approx H_q(x, 0, -\vec{\Delta}_\perp^2)
\end{aligned}
$$

Transverse Distribution

- separation of scales, $L_{TD/WP}$ is the transverse distribution/wave packet length scale

$$L_{TD} \sim \frac{1}{\left|\vec{\mathbf{\Delta}}_\perp\right|} \gg L_{WP} \gg \frac{1}{\sqrt{M^2 + P_z^2}} \qquad \Rightarrow \qquad L_{TD} \gg \frac{1}{\sqrt{M^2 + P_z^2}} \to 0$$

- transverse quark distribution

$$\int d^2 b_\perp \; e^{i\vec{\mathbf{\Delta}}_\perp \cdot \vec{b}_\perp} q(x, \vec{b}_\perp) = H_q(x, 0, -\vec{\mathbf{\Delta}}_\perp^2)$$

Lattice Calculation

I. Generalized Form Factors

II. Full QCD Calculation with SESAM

III. Hybrid Calculation with MILC

Definition of Generalized Form Factors

- unpolarized and polarized twist two operators from the light cone expansion

$$O_q^{\mu_1 \cdots \mu_n} = \overline{q} iD^{(\mu_1} \ldots iD^{\mu_{n-1}} \gamma^{\mu_n)} q$$

$$\tilde{O}_q^{\mu_1 \cdots \mu_n} = \overline{q} iD^{(\mu_1} \ldots iD^{\mu_{n-1}} \gamma^{\mu_n)} \gamma^5 q$$

- off-forward matrix elements of the twist two operators

$$\langle P', S' | O_q^{\mu_1 \ldots \mu_n} | P, S \rangle = \overline{U}(P', S') [\sum_{\substack{i=0 \\ \text{even}}}^{n-1} A_{ni}^q(t) K_{ni}^A(P', P) + \sum_{\substack{i=0 \\ \text{even}}}^{n-1} B_{ni}^q(t) K_{ni}^B(P', P) + \delta_{\text{even}}^n C_n^q(t) K_n^C(P', P)] U(P, S)$$

$$\langle P', S' | \tilde{O}_q^{\mu_1 \ldots \mu_n} | P, S \rangle = \overline{U}(P', S') [\sum_{\substack{i=0 \\ \text{even}}}^{n-1} \tilde{A}_{ni}^q(t) \tilde{K}_{ni}^A(P', P) + \sum_{\substack{i=0 \\ \text{even}}}^{n-1} \tilde{B}_{ni}^q(t) \tilde{K}_{ni}^B(P', P)] U(P, S)$$

Equivalence of Generalized Form Factors and Generalized Parton Distributions

- moments of generalized parton distributions (unpolarized example)

$$\int_{-1}^{1} dx\ x^{n-1} H_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} A^q_{n,2i}(t)(-2\xi)^{2i} + \mathrm{mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

$$\int_{-1}^{1} dx\ x^{n-1} E_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} B^q_{n,2i}(t)(-2\xi)^{2i} - \mathrm{mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

- transverse momentum transfer, $\xi \to 0$

$$\int_{-1}^{1} dx\ x^{n-1} H_q(x,0,t) = A^q_{n0}(t)$$

$$\int_{-1}^{1} dx\ x^{n-1} E_q(x,0,t) = B^q_{n0}(t)$$

Basic Properties of Generalized Form Factors

- moments of parton distributions - $\langle P|O_q^{\mu_1\cdots\mu_n}|P\rangle$ and $\langle P|\tilde{O}_q^{\mu_1\cdots\mu_n}|P\rangle$

$$A_{n0}^q(0) = \int_{-1}^{1} dx\ x^{n-1}\, q(x) \quad \text{and} \quad \tilde{A}_{n0}^q(0) = \int_{-1}^{1} dx\ x^{n-1}\, \Delta q(x)$$

- form factors - $\mathcal{O}_q^\mu = \overline{q}\gamma^\mu q$ and $\tilde{\mathcal{O}}_q^\mu = \overline{q}\gamma^\mu\gamma^5 q$

$$A_{10}^q(t) = F_1^q(t) \quad \text{and} \quad B_{10}^q(t) = F_2^q(t)$$

$$\tilde{A}_{10}^q(t) = G_A^q(t) \quad \text{and} \quad \tilde{B}_{10}^q(t) = G_P^q(t)$$

- quark angular momenta [1]

- transverse quark distributions [2]

$$\int_{-1}^{1} dx\ x^{n-1}\, q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{n0}^q(-\vec{\Delta}_\perp^2)$$

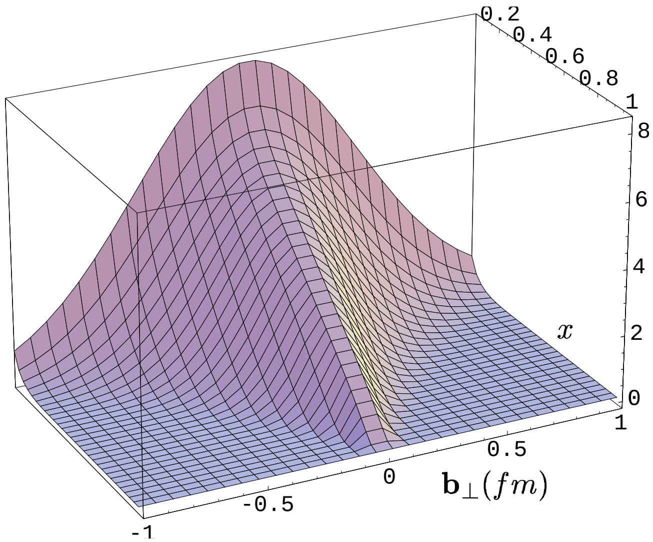
$$\int_{-1}^{1} dx\ x^{n-1}\, \Delta q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \tilde{A}_{n0}^q(-\vec{\Delta}_\perp^2)$$

[1] X. D. Ji hep-ph/9603249
[2] M. Burkardt hep-ph/0005108

Full QCD Calculation with Heavy Quarks

- SESAM gauge fields
- Wilson gluons and $N_F = 2$ Wilson fermions
- $a = 0.095$ fm, $L = 1.52$ fm
- $M_\pi = 753(10), 835(13), 895(15)$ MeV

Transverse Distributions



$$A^q_{n0}(-\vec{\Delta}^2_\perp) = \int d^2b_\perp \, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} \, q(x,\vec{b}_\perp)$$

$$\langle b^2_\perp \rangle^q_{(n)} = -4 \frac{A^{q\,\prime}_{n0}(0)}{A^q_{n0}(0)}$$

- at $x = 1$ a single quark carries all the momentum

$$\lim_{x\to 1} q(x,\vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

- higher moments A^q_{n0} weight $x \sim 1$ more heavily

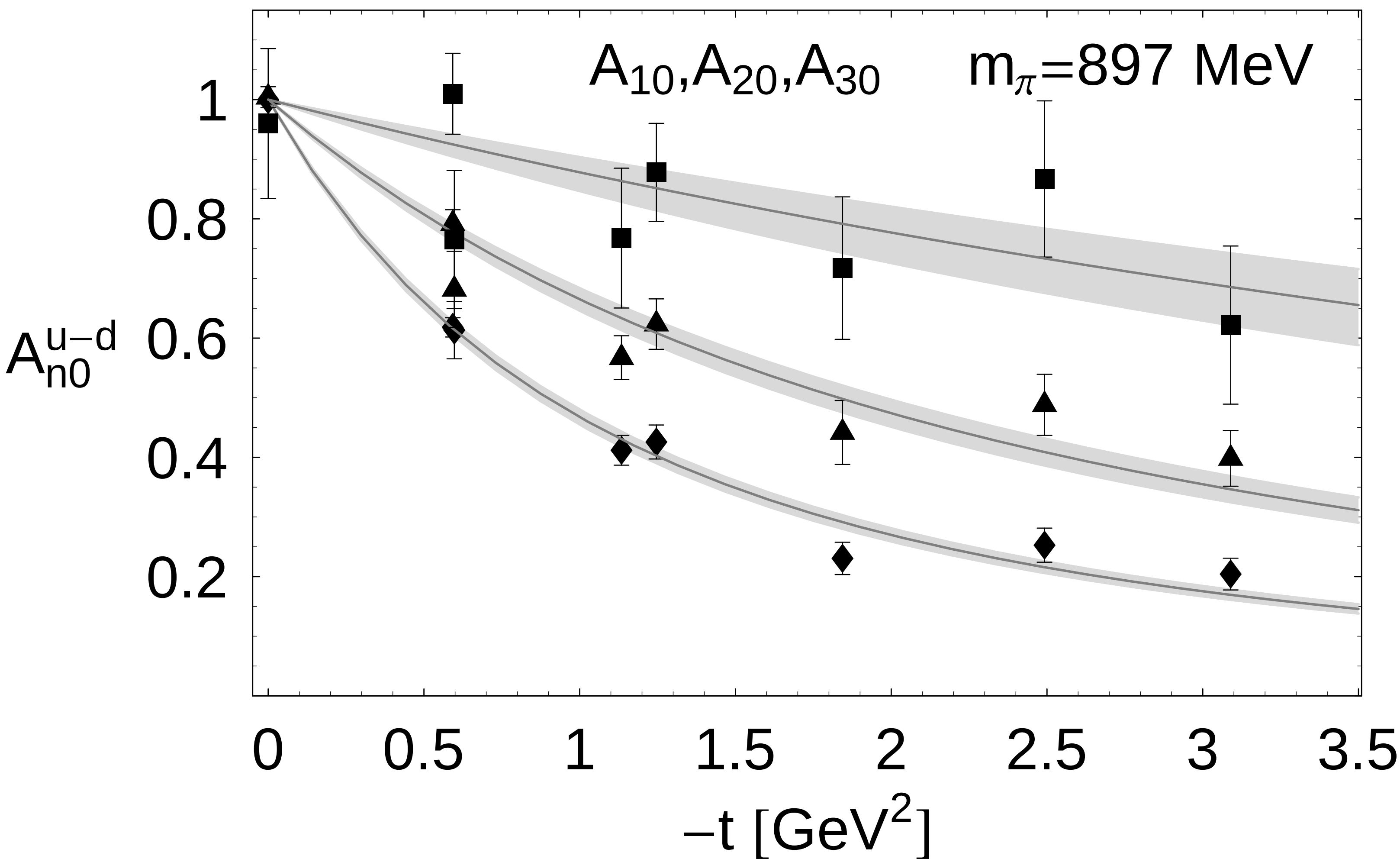
$$\lim_{n\to\infty} A^q_{n0}(t) \propto \int d^2b_\perp \, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \delta^2(\vec{b}_\perp) = \text{constant}$$

- slopes of A^q_{n0} should decrease as n increases

- A_{10}, A_{30}, $\tilde{A}_{20}$ measure $q - \overline{q}$ & $\tilde{A}_{10}$, $\tilde{A}_{30}$, A_{20} measure $q + \overline{q}$

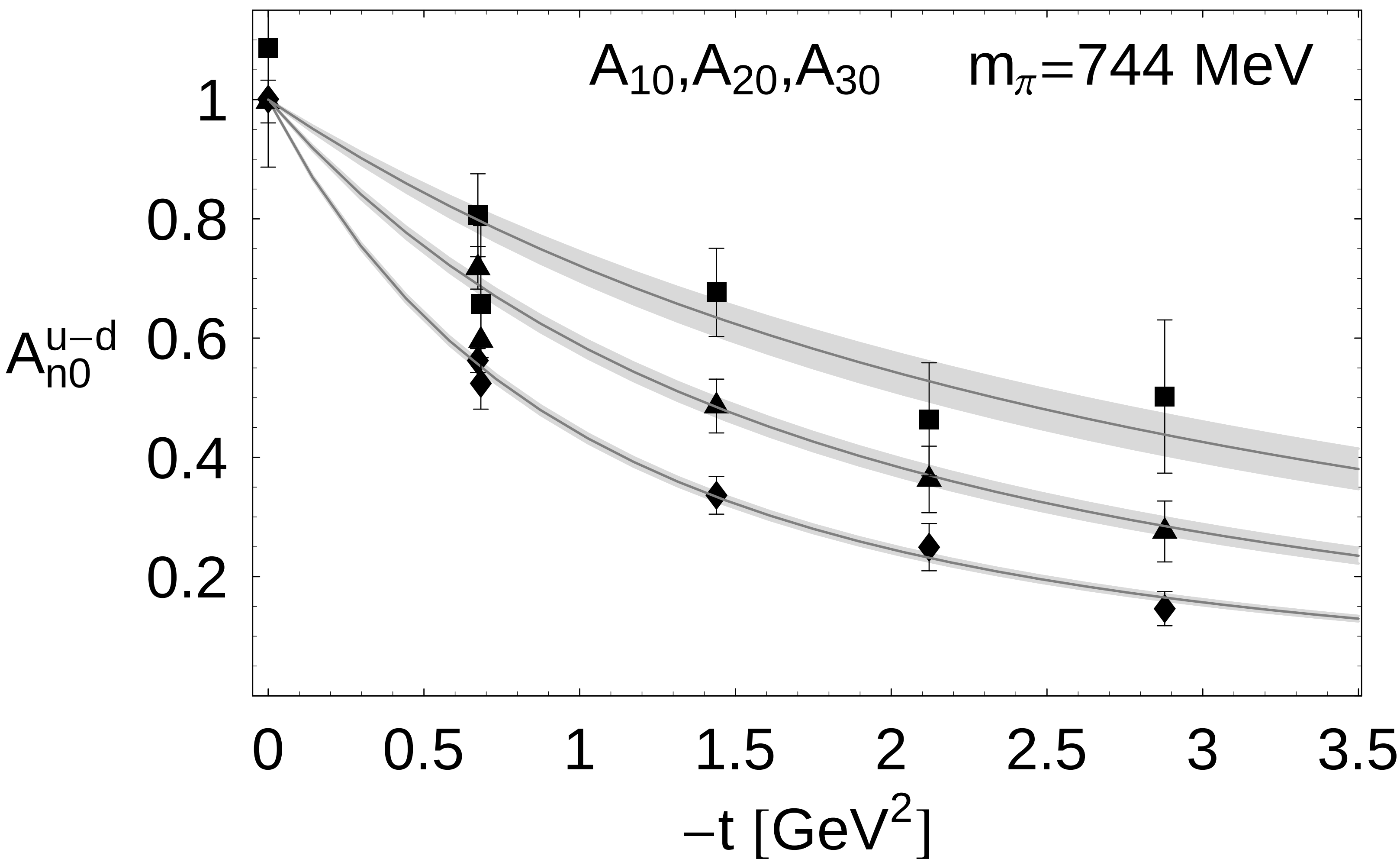
Transverse Distributions: $m_\pi = 897$ MeV

- slope of $A_{10}^{u-d} = -0.93 \pm 0.04\ (\text{GeV})^{-2}$
- slope of $A_{30}^{u-d} = -0.13 \pm 0.03\ (\text{GeV})^{-2}$ (factor of 7)



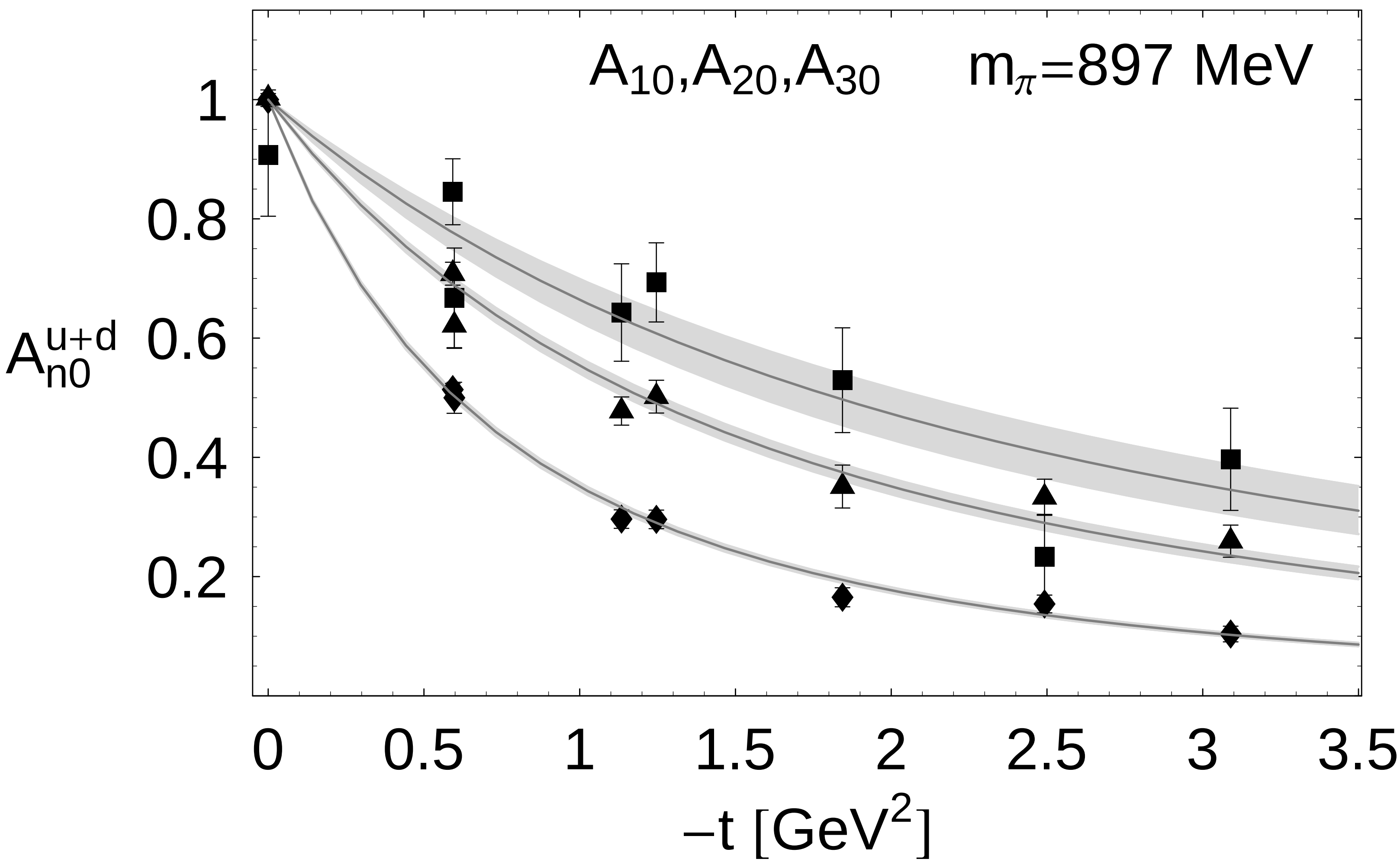
Transverse Distributions: Mass Dependence

- slope of $A_{10}^{u-d} = -1.02 \pm 0.03\ (\mathrm{GeV})^{-2}$
- slope of $A_{30}^{u-d} = -0.36 \pm 0.04\ (\mathrm{GeV})^{-2}$ (factor of 3)



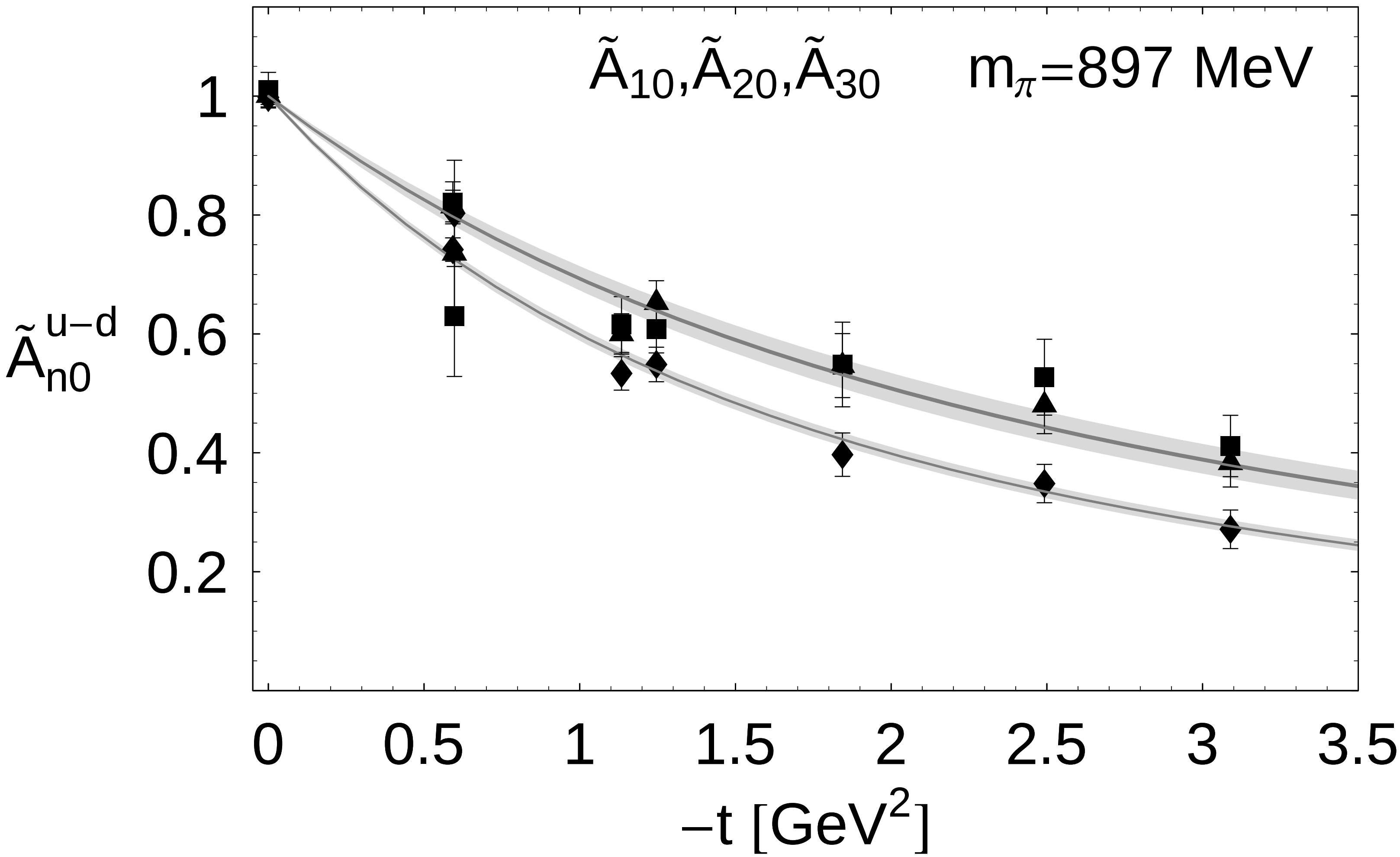
Transverse Distributions: Flavor Dependence

- slope of $A_{10}^{u+d} = -1.38 \pm 0.03\ (\mathrm{GeV})^{-2}$
- slope of $A_{30}^{u+d} = -0.45 \pm 0.07\ (\mathrm{GeV})^{-2}$ (factor of 3)



Transverse Distributions: Spin Dependence

- slope of $\tilde{A}_{10}^{u-d} = -0.58 \pm 0.02$ (GeV)$^{-2}$
- slope of $\tilde{A}_{30}^{u-d} = -0.40 \pm 0.03$ (GeV)$^{-2}$ (factor of 1.5)

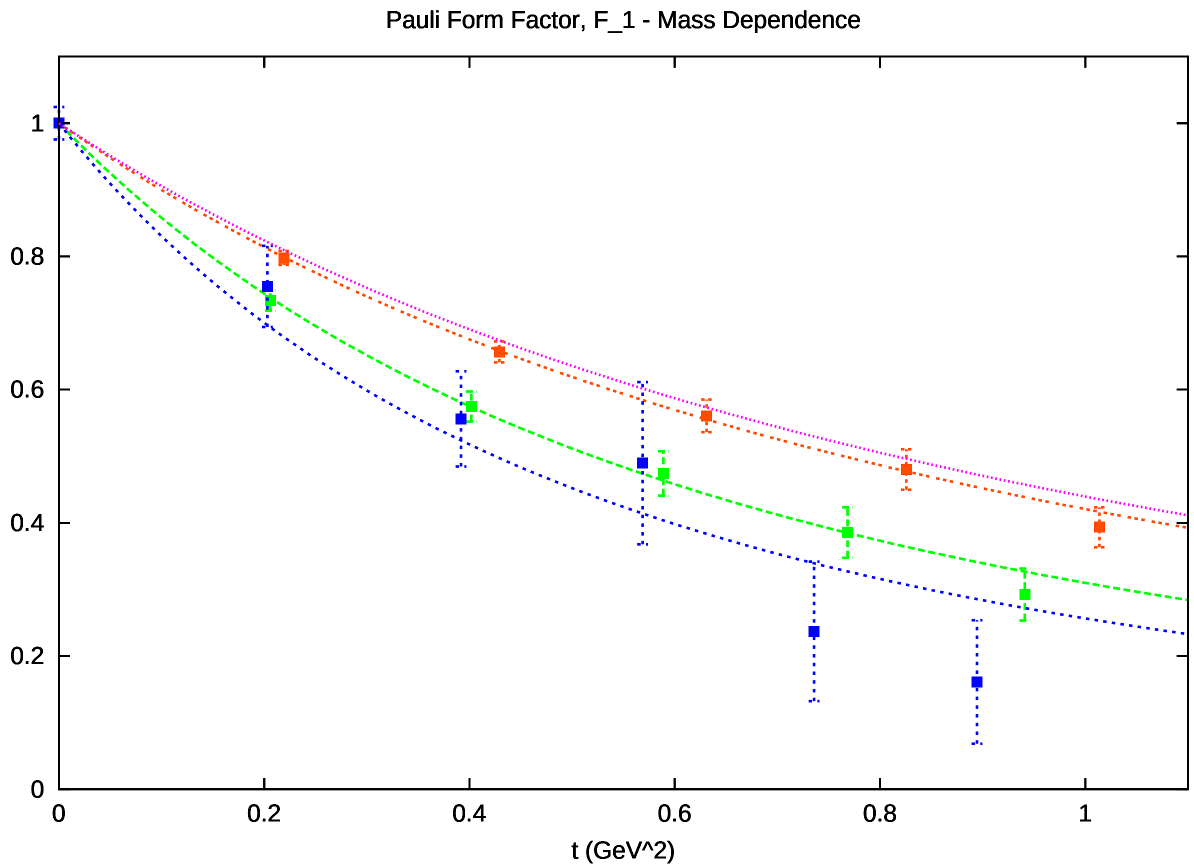


Hybrid Calculation with Lighter Quarks

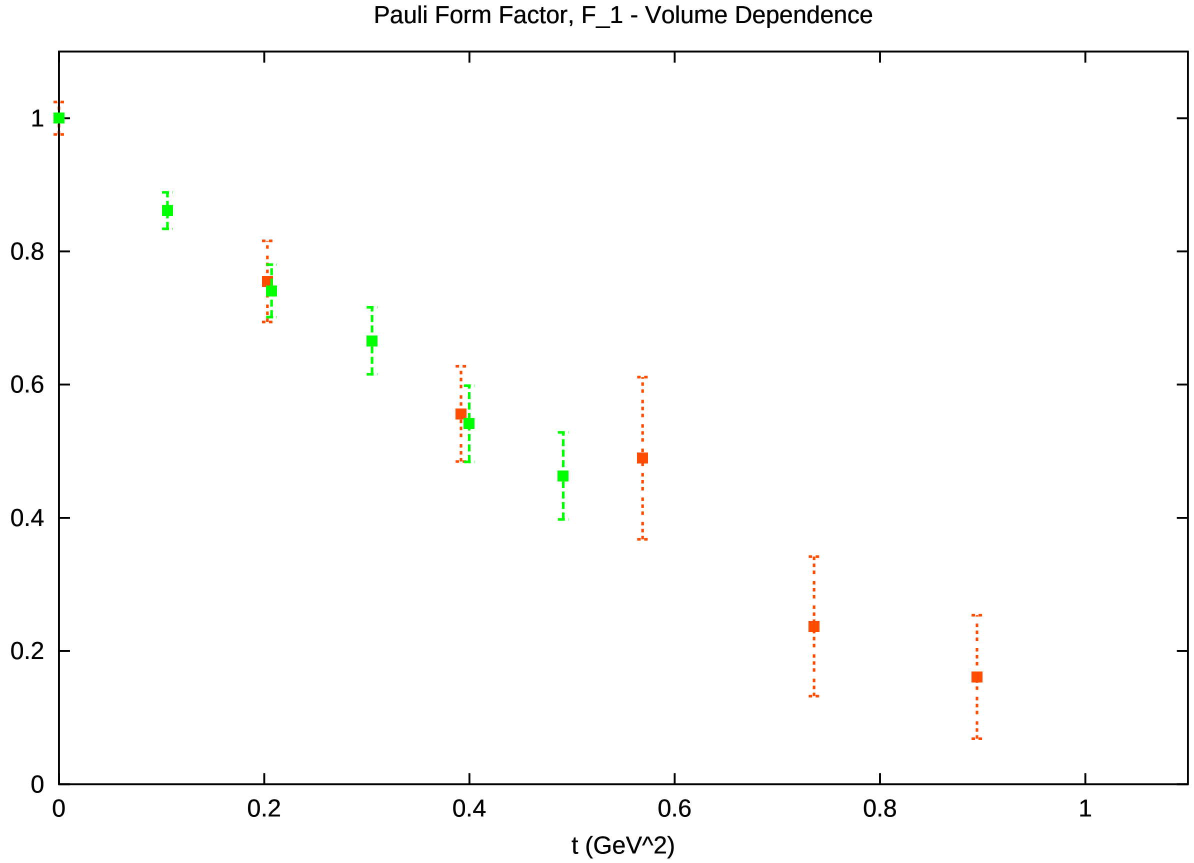
- asqtad staggered sea quarks - MILC
- domain wall valence quarks - $M = 1.7$ with HYP Smearing - $\alpha_1 = 0.75$, $\alpha_2 = 0.6$, $\alpha_3 = 0.3$

$am_{u/d}^{\text{asqtad}}$	L/a	a	L	m_π^{asqtad}	$am_{u/d}^{\text{DWF}}$	m_π^{DWF}	#
		fm	fm	MeV		MeV	
0.05	20	0.131	2.62	730(3)	0.0810	725(4)	107
0.03	20	0.132	2.64	564(2)	0.0478	570(3)	134
0.01	20	0.135	2.70	329(1)	0.0138	337(5)	104
0.01	28		3.78		0.0138	333(2)	131

F_1 Form Factor - Mass Dependence

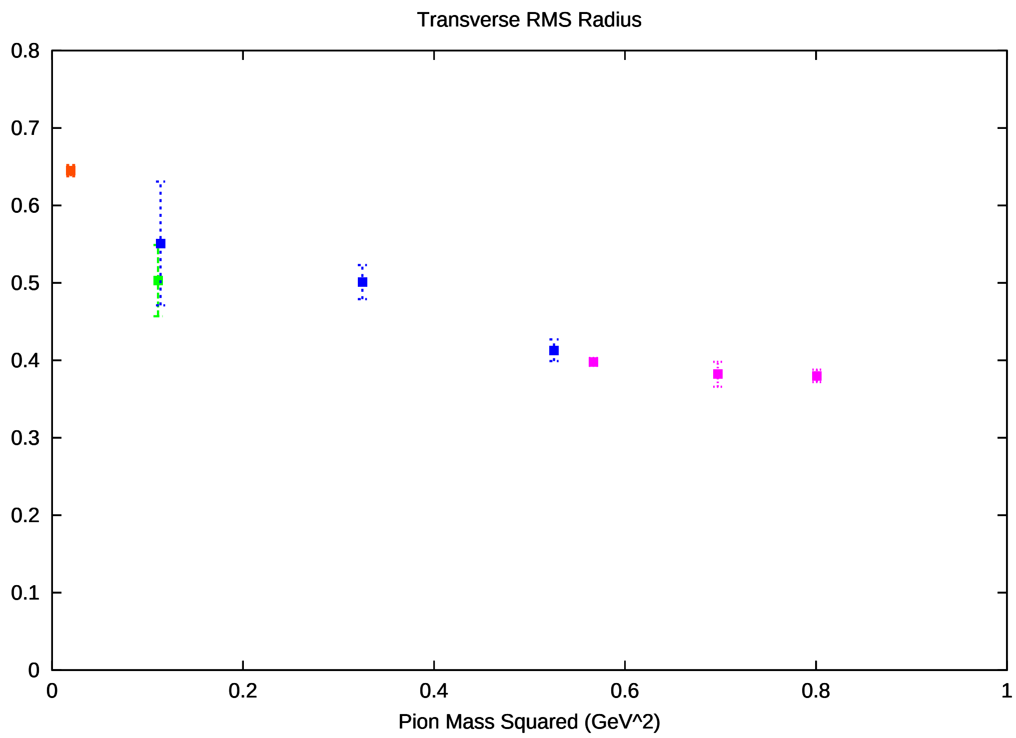


F_1 Form Factor - Volume Dependence



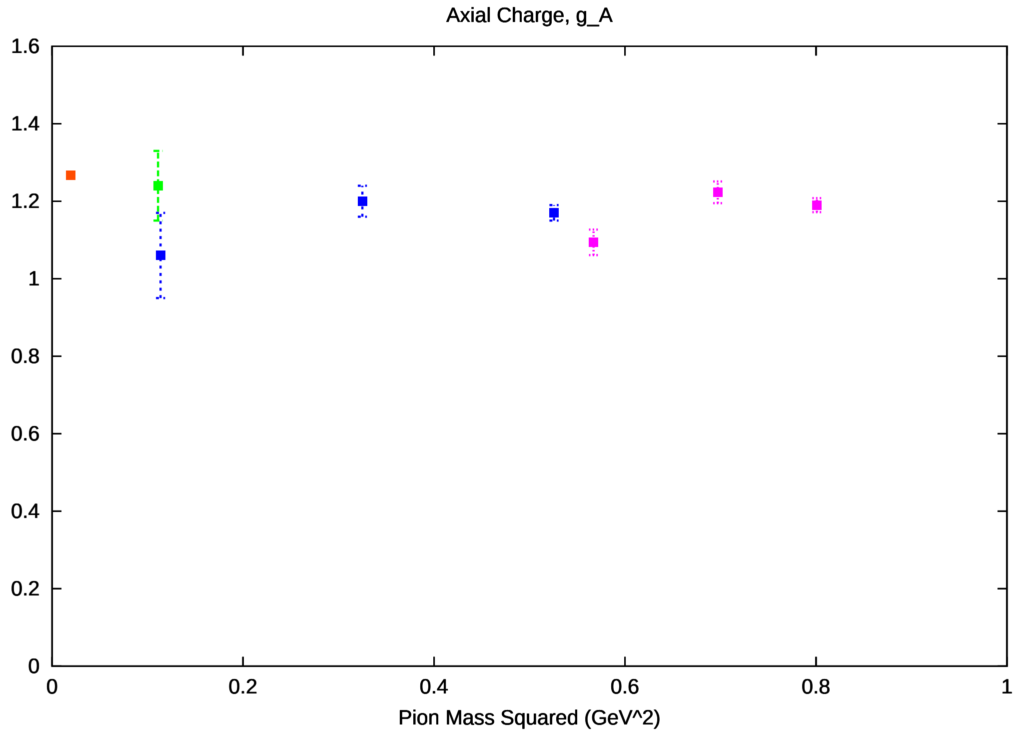
Transverse RMS Radius $\sqrt{r_\perp^2}$

m_π^{DWF}	L/a	Z_V	$\sqrt{r_\perp^2}$ (fm)
725	20	1.153(2)	0.413(14)
570	20	1.132(3)	0.501(22)
337	20	1.117(27)	0.551(80)
333	28	1.108(7)	0.503(46)



Axial Charge g_A

m_π^{DWF}	L/a	Z_A	Z_V	g_A
725	20	1.1282( )	1.153(2)	1.17(2)
570	20	1.1066(6)	1.132(3)	1.20(4)
337	20	1.0852(8)	1.117(27)	1.06(11)
333	28	1.0838(1)	1.108(7)	1.24(9)



Conclusions

- the transverse size of the nucleon $\langle r_{\perp}^2 \rangle$, in the heavy pion world, shows a significant dependence on the longitudinal momentum $\langle x \rangle$

- g_A and F_1 patch smoothly between the SESAM and MILC lattices and approach the experimental results in the lighter pion world

- continuing with the complete calculation of the generalized form factors with additional quark masses and extended statistics