Transverse Structure of Nucleon Parton Distributions

Dru B. Renner

Lattice Hadron Physics Collaboration Meeting

January 17, 2004

Generalized parton distributions determine the angular momentum decomposition of the nucleon and the transverse distribution of partons in the nucleon. Additionally, in particular limits they reduce to form factors and ordinary parton distributions. I will present our full QCD lattice calculations of the quark helicity and orbital angular momentum in the nucleon, the asymptotic scaling ratio F_2/F_1, and the transverse distribution of quarks in the nucleon. Additionally, I will present results from an exploratory hybrid calculation with light quarks.

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http://talks.drubryantrenner.org/lhpc_1-8-04.pdf

In Short

- generalized parton distributions - 3D distribution of quarks in a mixed representation - 2 transverse coordinates and 1 longitudinal momentum

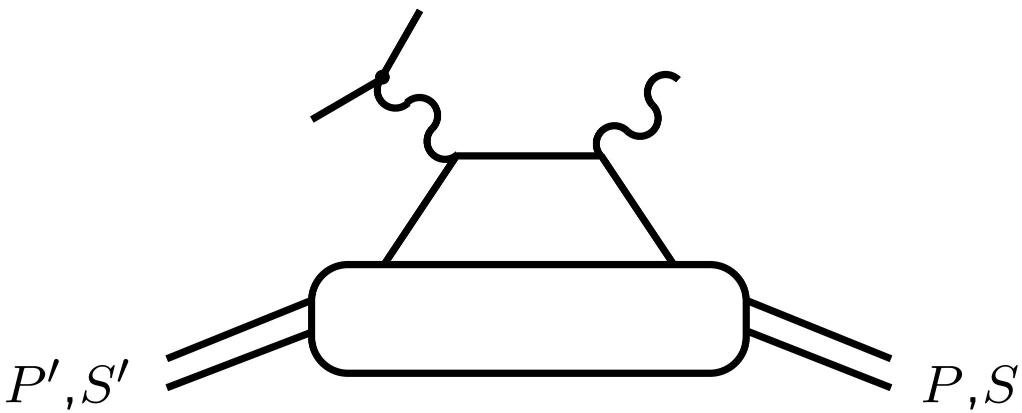
- generalized parton distributions - decomposition of nucleon spin into quark helicity, quark orbital, and gluon contributions

- form factors - asymptotic Q^2 limit of F_2/F_1

- ordinary parton distributions - exploratory hybrid calculation with light quarks

Generalized Parton Distributions

- deeply virtual Compton scattering, $lN \to lN\gamma$, exclusive



$$\overline{P} = \tfrac{1}{2}\,(P' + P)$$

$$\xi = -\frac{\Delta^+}{2\overline{P}^+}$$

- off-forward matrix element of light-cone quark correlator

$$\left\langle P', S' \left| \mathcal{O}_q(x, \vec{0}_\perp) \right| P, S \right\rangle =$$

$$\frac{1}{2\overline{P}^+}\overline{U}(P', S') \left(\gamma^+ H(x, \xi, t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M} E(x, \xi, t) \right) U(P, S)$$

- meson production, Compton scatering

GPD reviews: Ji hep-ph/9807358, Diehl hep-ph/0307382

Generalized Parton Distributions

- familiar limits: parton distributions & form factors

$$H_q(x,0,0) = q(x)$$

$$\sum_q e_q \int dx\ H_q(x,\xi,t) = F_1(t)$$

$$\sum_q e_q \int dx\ E_q(x,\xi,t) = F_2(t)$$

- quark angular momentum [1]

$$J_q = \frac{1}{2}\int dx\ x\left(H_q(x,0,0) + E_q(x,0,0)\right)$$

- transverse quark distribution in the infinite momentum frame [2]

$$\int d^2b_\perp\ e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} q(x,\vec{b}_\perp) = H_q(x,0,-\vec{\Delta}_\perp^2)$$

[1] Ji hep-ph/9603249
[2] Burkardt hep-ph/0005108

Moments of Generalized Parton Distributions

- generalized parton distribtuions

$$\left\langle P', S' \left| \mathcal{O}_q(x) \right| P, S \right\rangle \;=$$
$$\frac{1}{2\overline{P}^+}\overline{U}(P', S')\left(\gamma^+ H_q(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M}E_q(x,\xi,t)\right)U(P,S)$$

$$\mathcal{O}_q(x,\vec{b}_\perp) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-}\,\overline{q}\left(-\frac{y^-}{2},\vec{b}_\perp\right)\gamma^+\,q\left(\frac{y^-}{2},\vec{b}_\perp\right)$$

- light-cone expansion & tower of twist 2 operators

$$\mathcal{O}_q^{\mu_1\mu_2\dots\mu_n} = \overline{q}\gamma^{\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}q$$

- moments of generalized parton distribtuions

$$\left\langle P' \middle| \mathcal{O}^{\mu_1\mu_2\dots\mu_n} \middle| P \right\rangle \sim \int dx\, x^{n-1} H_q(x,\xi,t) \quad \& \quad \int dx\, x^{n-1} E_q(x,\xi,t)$$

Generalized Form Factors

- $n+1$ generalized form factors

$$\left\langle P' | \mathcal{O}_q^{\mu_1 \mu_2 \cdots \mu_n} | P \right\rangle \sim A_{ni}^q(t),\ B_{ni}^q(t),\ C_n^q(t)$$

- moments of parton distributions

$$\left\langle x^{n-1} \right\rangle_q = A_{n0}^q(0)$$

- form factors

$$F_1(t) = \sum_q e_q A_{10}^q(t) \qquad F_2(t) = \sum_q e_q B_{10}^q(t)$$

- quark angular momentum

$$J_q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$$

Ji, Lebed hep-ph/0012160

Generalized Form Factors

- moments of generalized parton distributions

$$\int_{-1}^{1} dx\ x^{n-1} H_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} A^q_{n,2i}(t)(-2\xi)^{2i} + \mathrm{mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

$$\int_{-1}^{1} dx\ x^{n-1} E_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} B^q_{n,2i}(t)(-2\xi)^{2i} - \mathrm{mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

- transverse momentum transfer, $\xi \to 0$

$$\int_{-1}^{1} dx\ x^{n-1} H_q(x,0,t) = A^q_{n0}(t)$$

$$\int_{-1}^{1} dx\ x^{n-1} E_q(x,0,t) = B^q_{n0}(t)$$

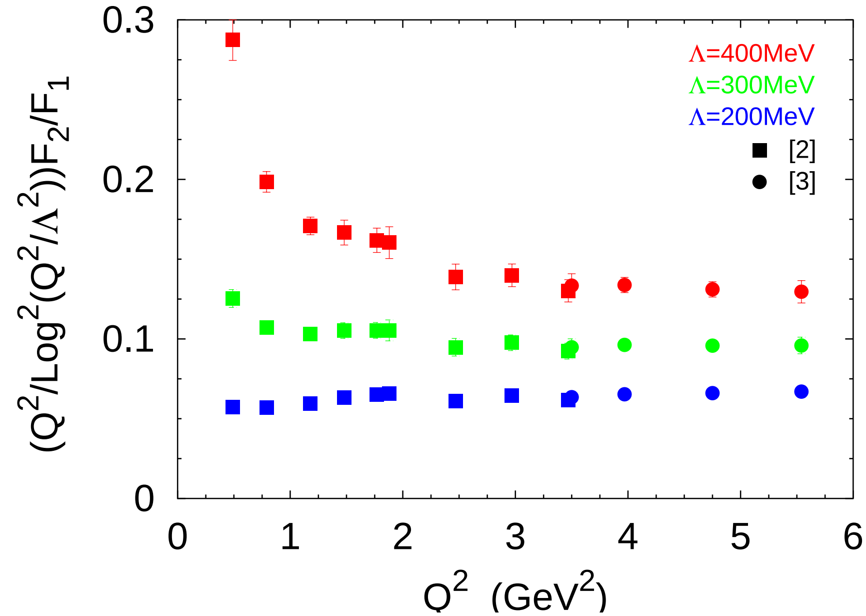
Simulation Parameters

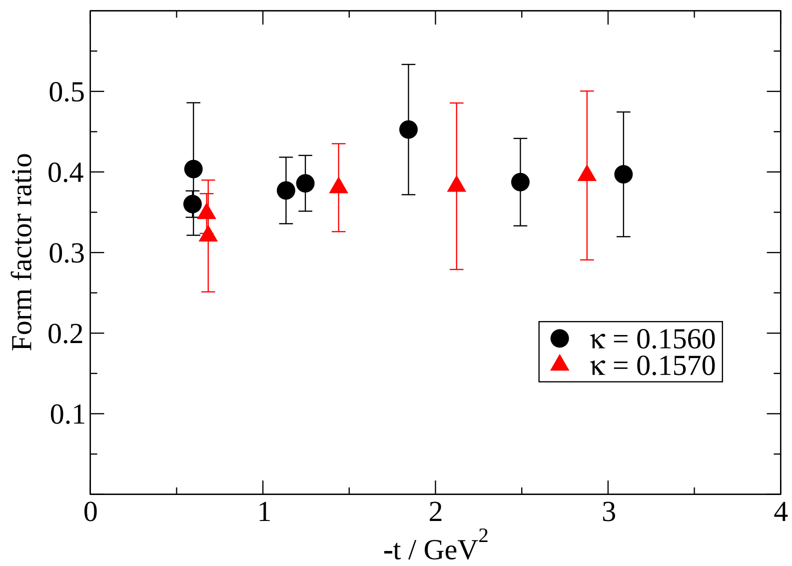
- SESAM gauge fields
- $N_F = 2$ Wilson fermions
- $a = 0.092$ fm, $L = 1.48$ fm, $16^3 \times 32$
- O(200) gluon configurations
- $M_\pi = 744,\ 831,\ 897$ MeV

Form Factor Ratios

- perturbative QCD & quark counting rules

$$\frac{Q^2 F_2(Q^2)}{\log^2(Q^2/\Lambda^2) F_1(Q^2)} \sim \text{constant}$$



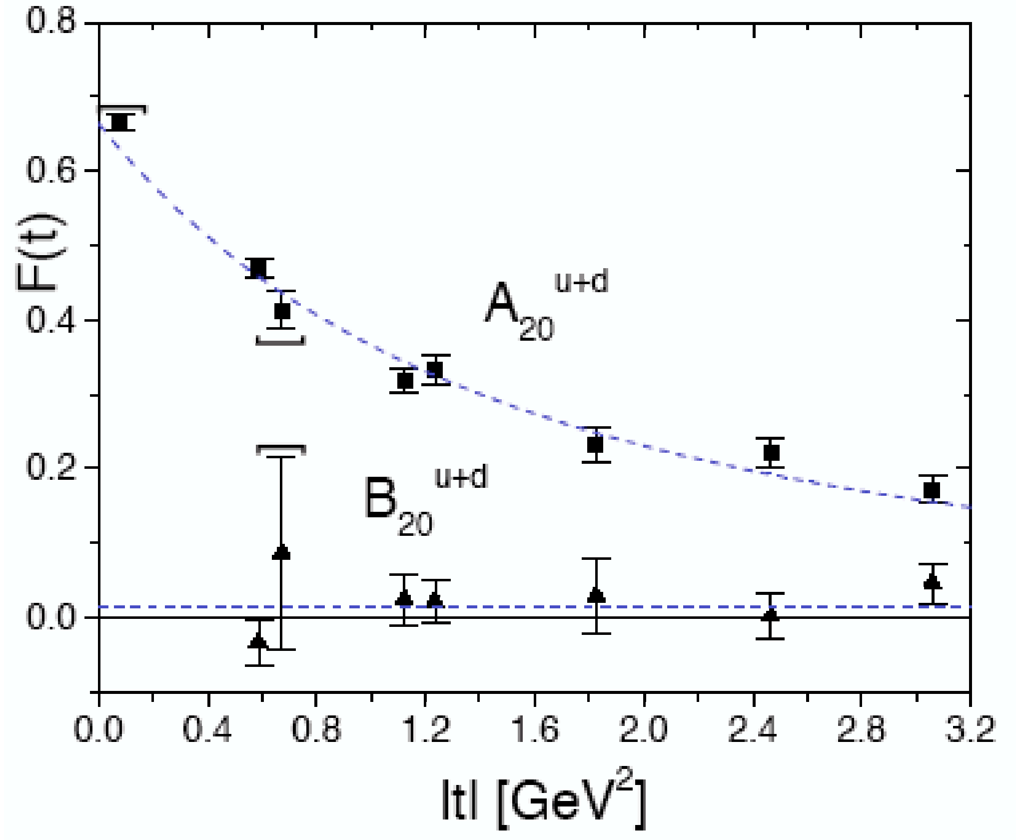


- note normalization difference: compare right plot with 2.79 times left plot

Belitsky, Ji, Yuan hep-ph/0212351

Quark Angular Momentum: $m_\pi = 897\text{MeV}$

- $\frac{1}{2}\Delta\Sigma_{u+d} = \frac{1}{2}\tilde{A}_{10}^{u+d}(0) = \frac{1}{2}\left(\langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d}\right) \sim \frac{1}{2}0.682(18)$
- $J_{u+d} = \frac{1}{2}\left(A_{20}^{u+d}(0) + B_{20}^{u+d}(0)\right) \sim \frac{1}{2}0.675(7)$
- $L_{u+d} = J_{u+d} - \frac{1}{2}\Delta\Sigma_{u+d} \sim 0 \quad \Rightarrow \quad J_g = \frac{1}{2}0.32$



QCDSF hep-ph/0304249; Mathur hep-ph/9912289; Gadiyak hep-lat/0112040

Transverse Structure

$$H_q(x, 0, -\vec{\Delta}_\perp^2) = \int d^2 b_\perp \; e^{i \vec{\Delta}_\perp \cdot \vec{b}_\perp} q(x, \vec{b}_\perp)$$

$$A_{n0}^q(t) = \int_{-1}^{1} dx \; x^{n-1} H_q(x, 0, t)$$

- at $x = 1$ a single quark carries all the momentum

$$\lim_{x \to 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp) \qquad H_q(1, 0, t) = \text{constant}$$

- higher moments A_{n0}^q weight $x \sim 1$ more heavily

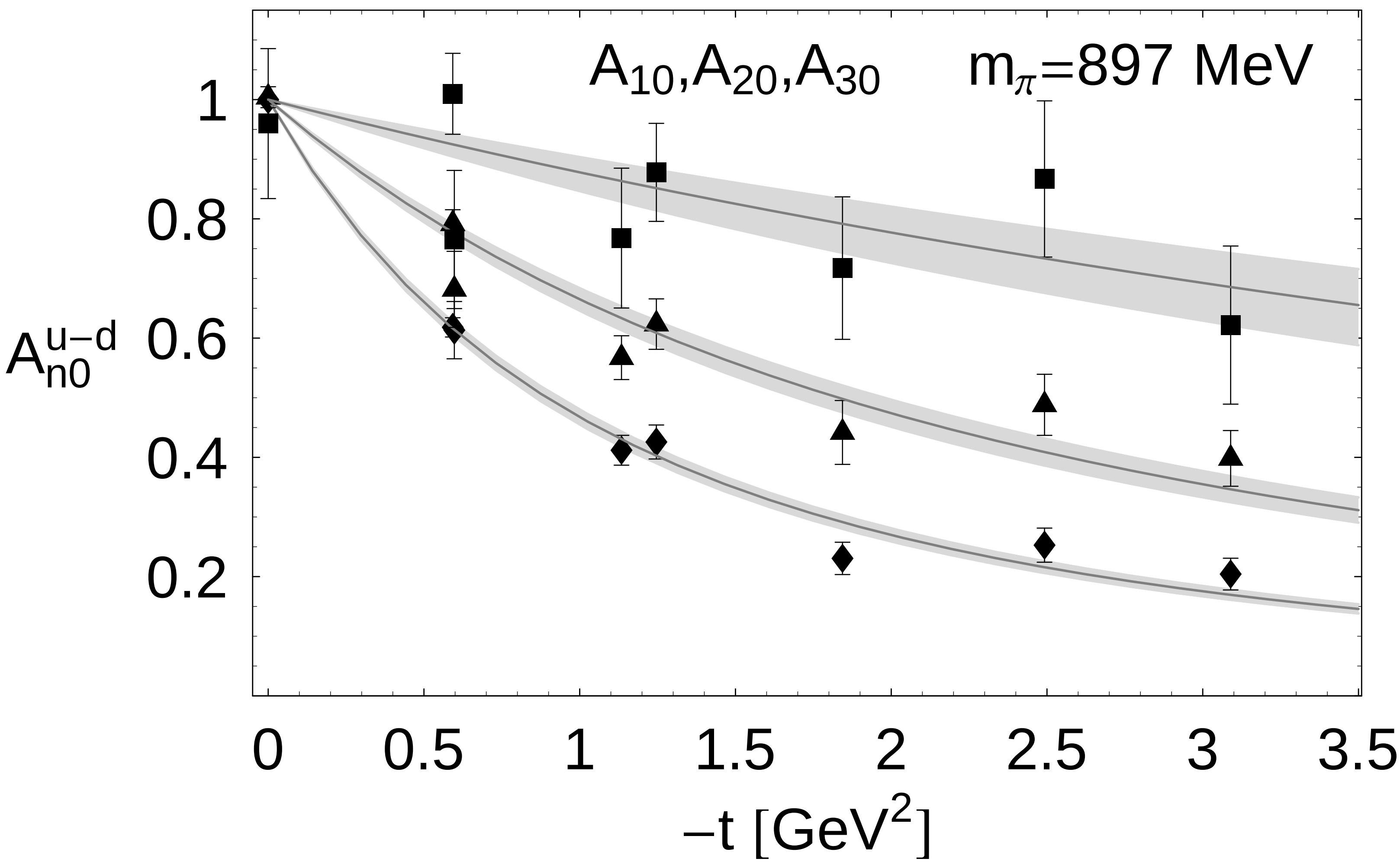
$$\lim_{n \to \infty} A_{n0}^q(t) \propto H_q(1, 0, t)$$

- slopes of A_{n0}^q should decrease as n increases

- A_{10}, A_{30}, $\tilde{A}_{20}$ measure $q - \overline{q}$ & $\tilde{A}_{10}$, $\tilde{A}_{30}$, A_{20} measure $q + \overline{q}$

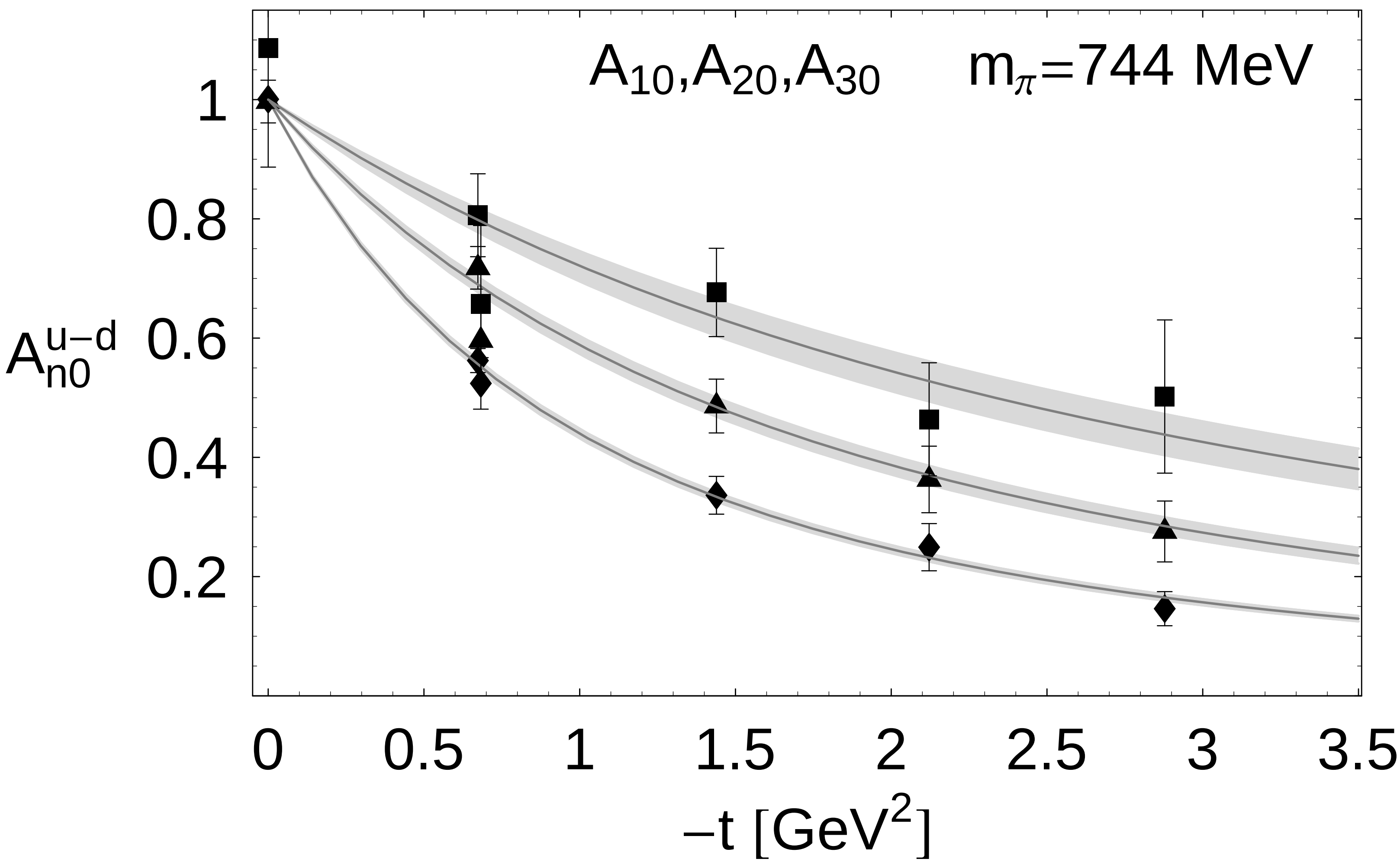
Transverse Structure: $m_\pi = 897$MeV

- slope of $A_{10}^{u-d} = -0.93 \pm 0.04$ (GeV)$^{-2}$
- slope of $A_{30}^{u-d} = -0.13 \pm 0.03$ (GeV)$^{-2}$ (factor of 7)



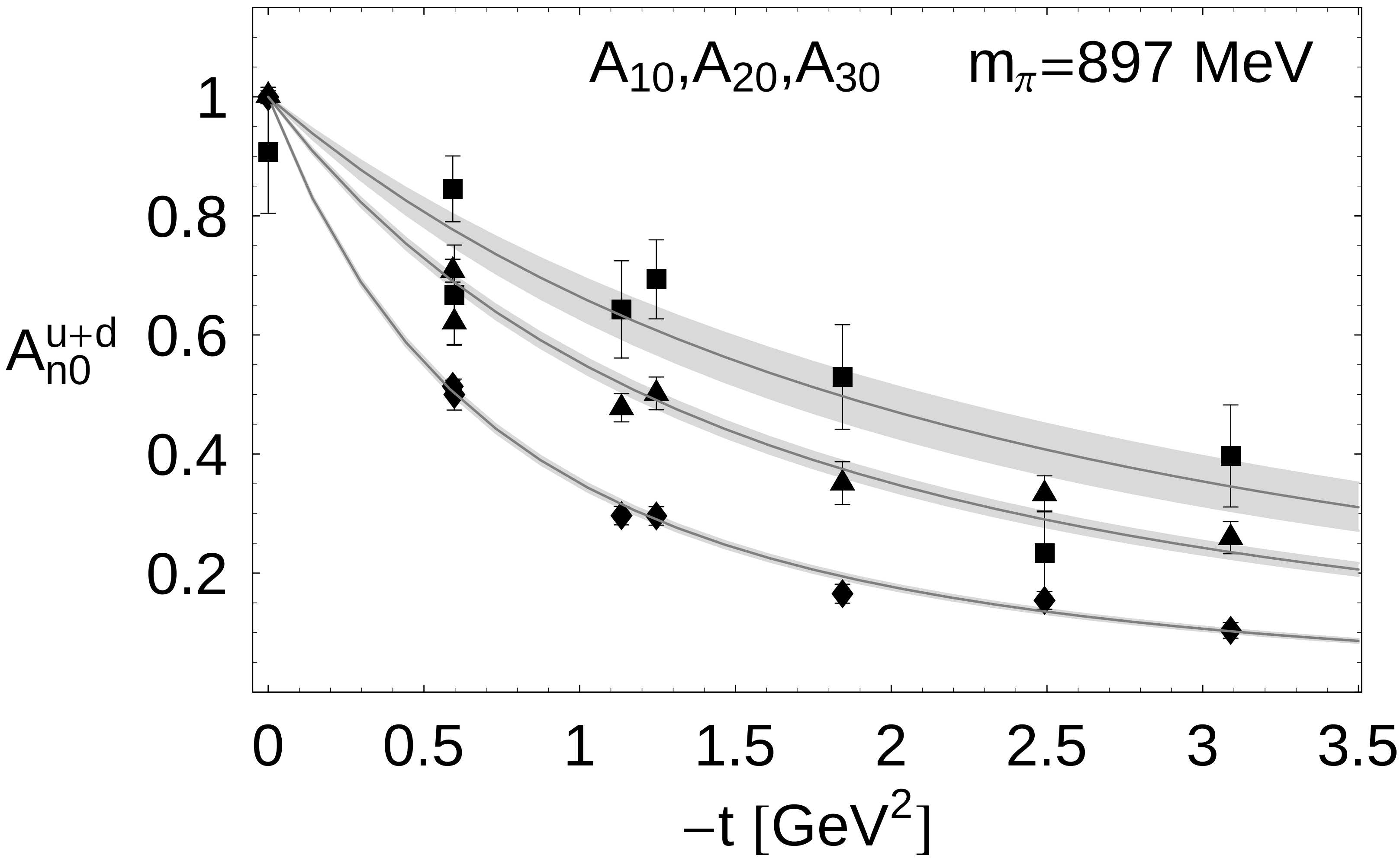
Transverse Structure: $m_\pi = 744\text{MeV}$

- slope of $A_{10}^{u-d} = -1.02 \pm 0.03\ (\text{GeV})^{-2}$
- slope of $A_{30}^{u-d} = -0.36 \pm 0.04\ (\text{GeV})^{-2}$ (factor of 3)



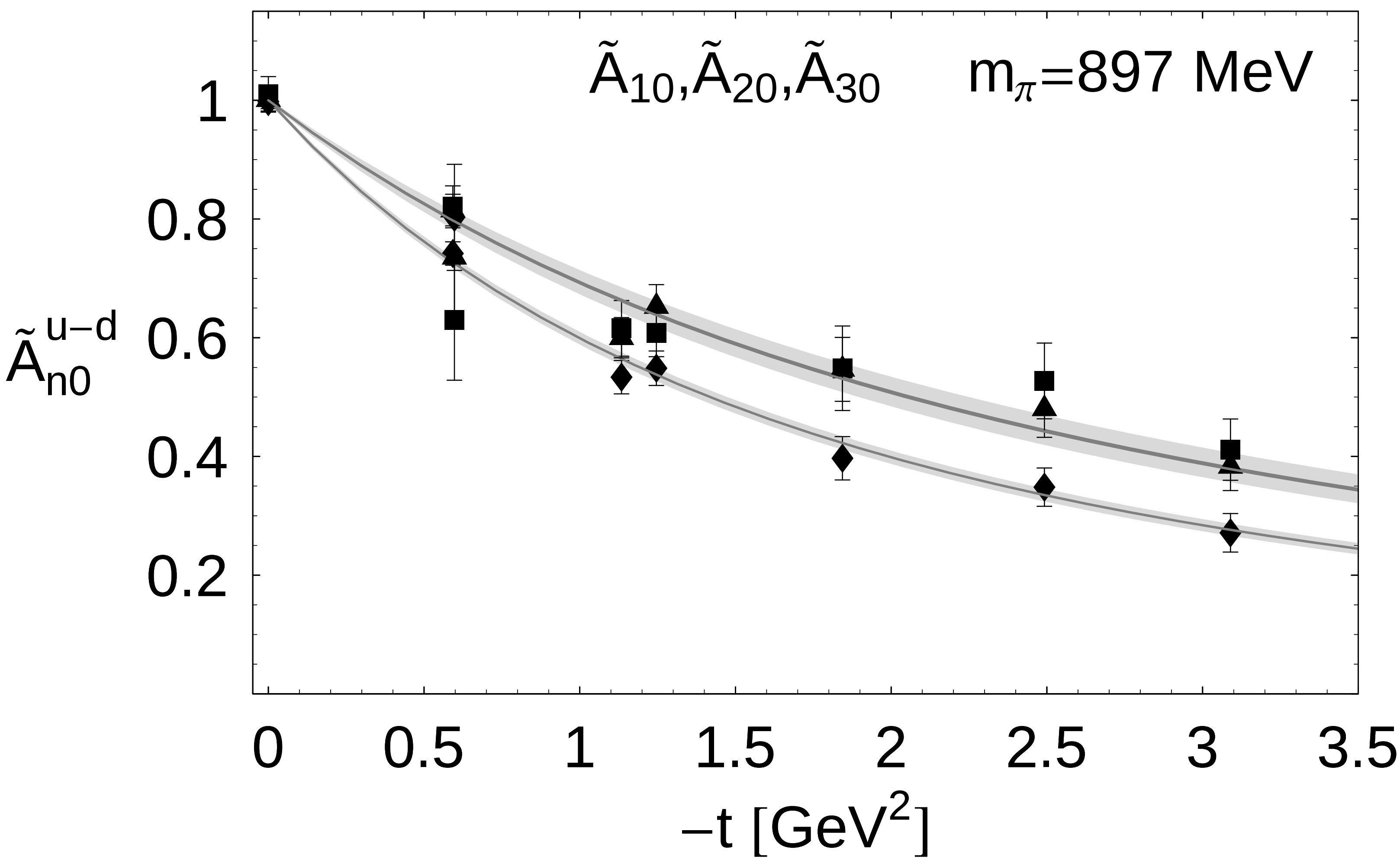
Transverse Structure: Flavor Dependence

- slope of $A_{10}^{u+d} = -1.38 \pm 0.03\ (\text{GeV})^{-2}$
- slope of $A_{30}^{u+d} = -0.45 \pm 0.07\ (\text{GeV})^{-2}$ (factor of 3)



Transverse Structure: Spin Dependence

- slope of $\tilde{A}_{10}^{u-d} = -0.58 \pm 0.02\ (\mathrm{GeV})^{-2}$
- slope of $\tilde{A}_{30}^{u-d} = -0.40 \pm 0.03\ (\mathrm{GeV})^{-2}$ (factor of 1.5)



Transverse Structure

- transverse rms radius

$$\left\langle b_\perp^2 \right\rangle_x = \frac{\int d^2b_\perp \, b_\perp^2 \, q(x,\vec{b}_\perp)}{\int d^2b_\perp \, q(x,\vec{b}_\perp)}$$

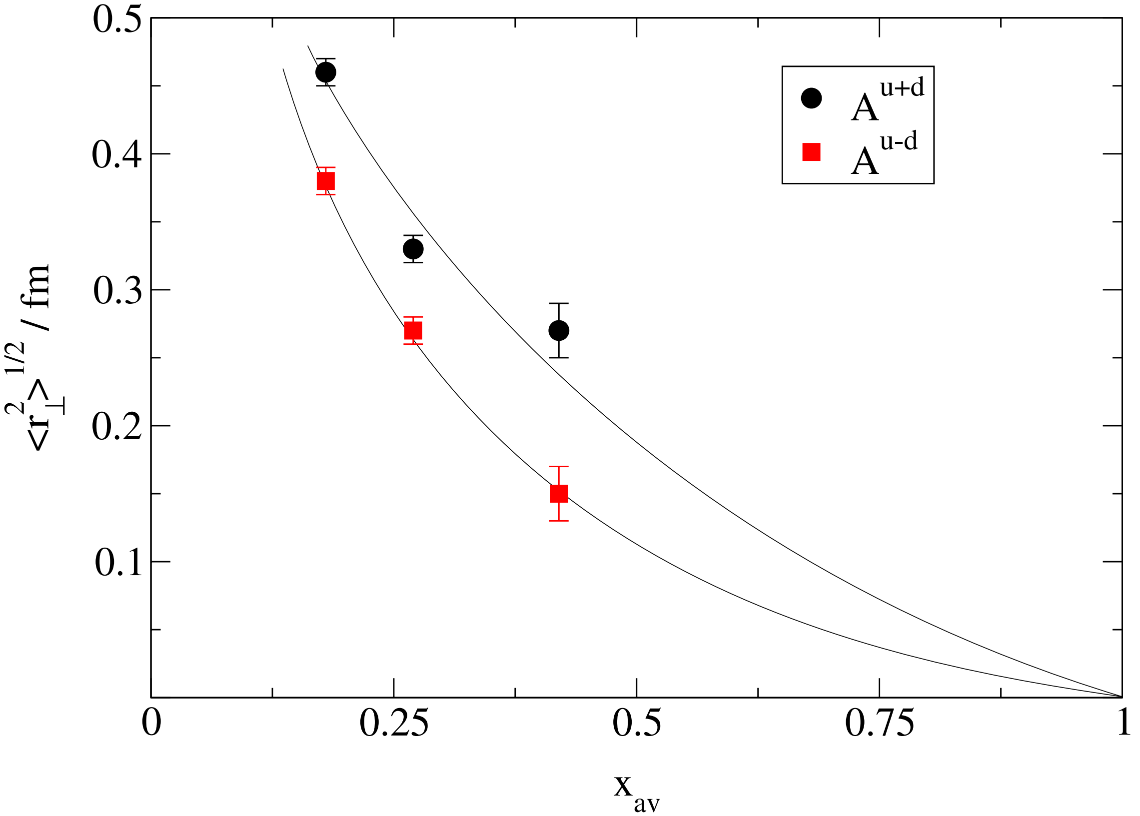
- smeared transverse rms radius

$$\frac{A'_{n0}(0)}{A_{n0}(0)} = -\frac{1}{4}\left\langle b_\perp^2 \right\rangle_{(n)} \qquad \left\langle b_\perp^2 \right\rangle_{(n)} = \frac{\int d^2b_\perp \, b_\perp^2 \, \int_{-1}^{1} dx \, x^{n-1} q(x,\vec{b}_\perp)}{\int d^2b_\perp \, \int_{-1}^{1} dx \, x^{n-1} q(x,\vec{b}_\perp)}$$

- the average x in $\left\langle b_\perp^2 \right\rangle_{(n)}$

$$x_{\text{av}}^{(n)} = \frac{\int d^2b_\perp \, \int_{-1}^{1} dx \, x \cdot x^{n-1} q(x,\vec{b}_\perp)}{\int d^2b_\perp \, \int_{-1}^{1} dx \, x^{n-1} q(x,\vec{b}_\perp)} = \frac{< x^n >}{< x^{n-1} >}$$

Smeared Transverse Structure



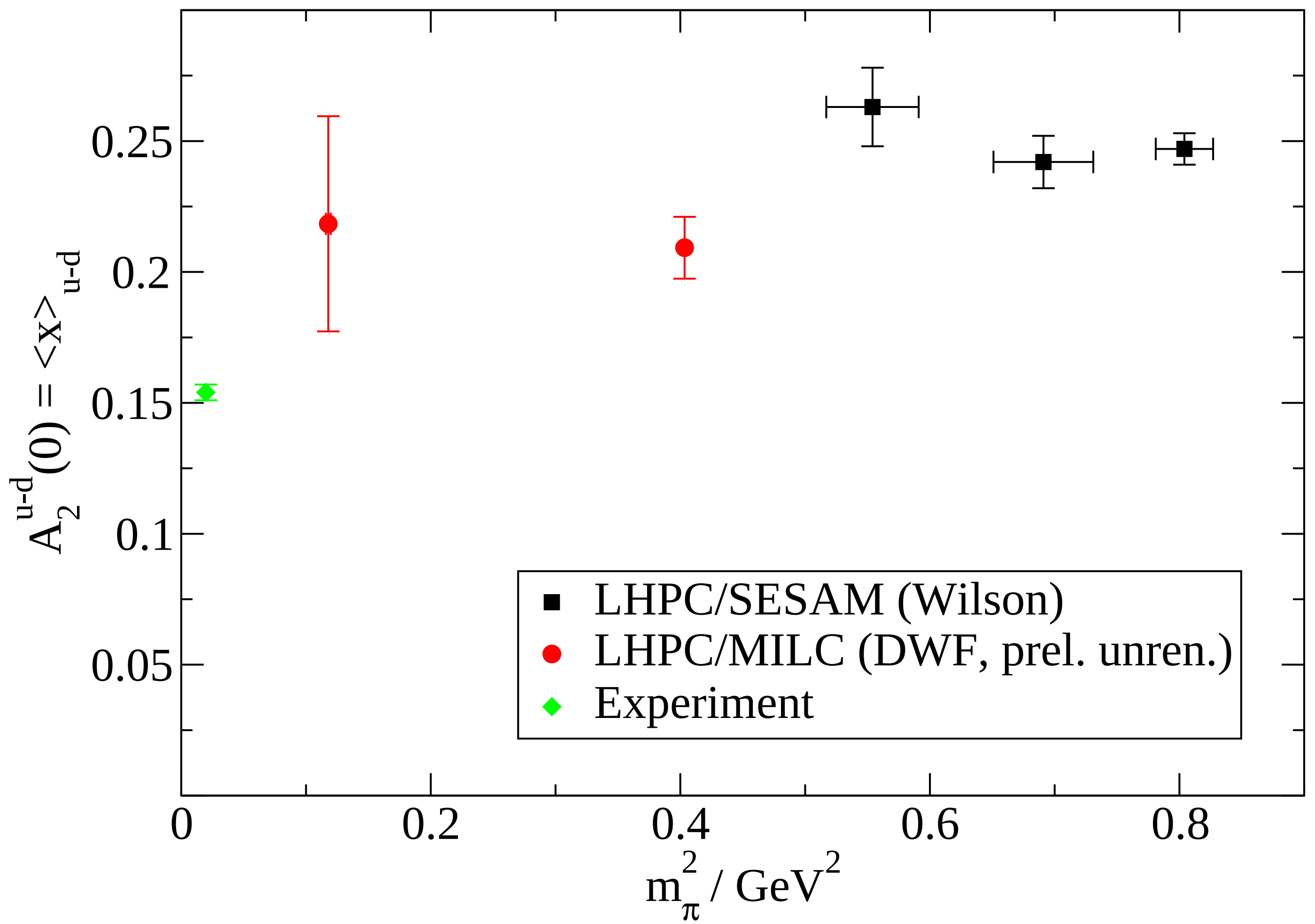
- charge radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{\text{charge}}} = 0.42$ fm (0.72 fm from experiment), no pion cloud
- non-singlet radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{u-d}^{(1)}} = 0.38$ fm & $\sqrt{\left\langle b_\perp^2 \right\rangle_{u-d}^{(3)}} = 0.15$ fm, 61% decrease
- singlet radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{u+d}^{(1)}} = 0.46$ fm & $\sqrt{\left\langle b_\perp^2 \right\rangle_{u+d}^{(3)}} = 0.27$ fm, 41% decrease

Current Work: Chiral Limit

- hybrid calculation: Asqtad staggered sea quarks (MILC) & domain wall valence quarks with HYP smearing
- $a = 0.13$ fm, $L = 2.6$ fm, $20^3 \times 32$
- $M_\pi = 343, 635$ MeV
- O(100) gluon configurations
- extended statistics, intermediate masses, volume dependence
- no renormalization factors for this action yet

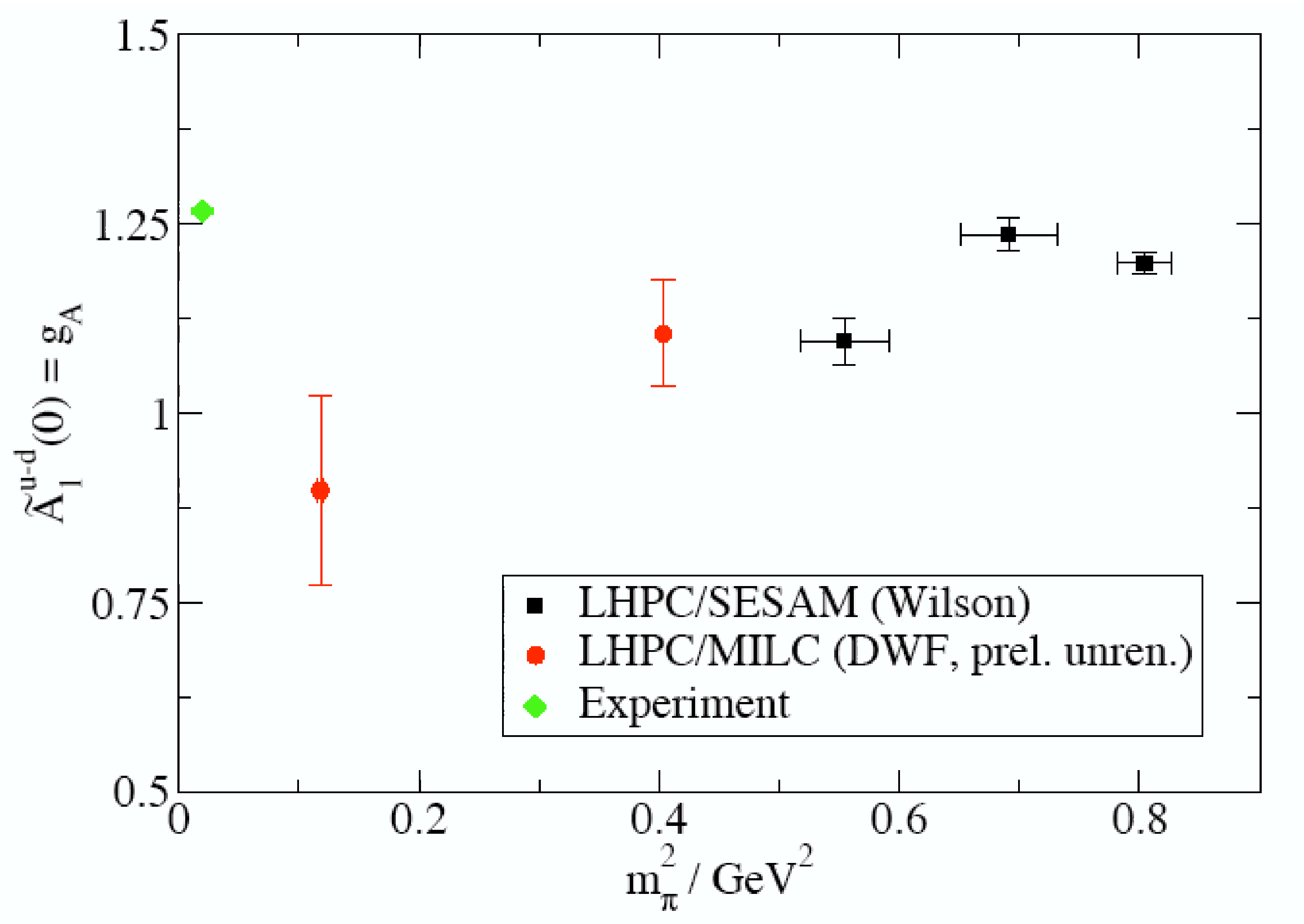
Moments of Parton Distributions

- $\langle x \rangle_{u-d}$ momentum fraction



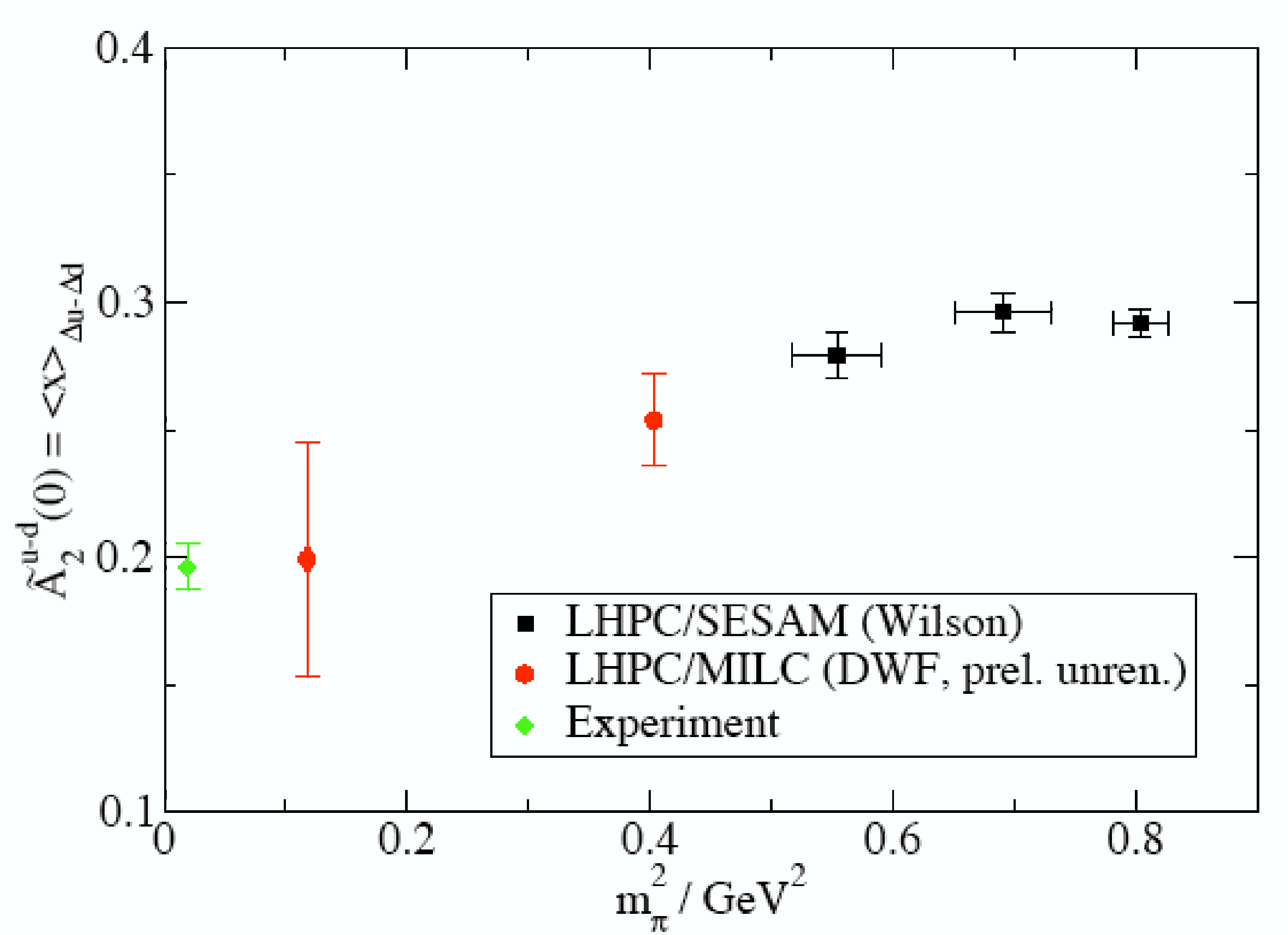
Moments of Parton Distributions

- $\langle 1 \rangle_{\Delta u - \Delta d}$ axial charge



Moments of Parton Distributions

- $\langle x \rangle_{\Delta u - \Delta d}$



Summary

- generalized parton distributions:
  (1) contain form factors and ordinary parton distributions
  (2) determine the spin decomposition of the nucleon
  (3) measure Fourier transform of transverse quark distribution

- in heavy pion world: $1/2 = 1/2\ (0.68 + 0.0 + 0.32)$

- observed scaling in F_2/F_1

- observed significant dependence of slopes of generalized form factors A_{n0} on n indicating a significant variation of transverse size with x

- exploratory hybrid calculation with light quarks