Nucleon Structure with Domain Wall Valence Quarks and Improved Staggered Sea Quarks

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Hybrid calculations with domain wall valence quarks and improved staggered sea quarks are being used to explore nucleon structure in full QCD in the light pion regime. Initial results for moments of structure functions, form factors, and generalized form factors will be presented and compared with previous results in the heavy pion regime.

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http://talks.drubryantrenner.org/lattice04.pdf

Definition of Generalized Form Factors

- unpolarized and polarized twist two operators

$$O_q^{\mu_1\cdots\mu_n} = \overline{q} iD^{(\mu_1} \ldots iD^{\mu_{n-1}}\gamma^{\mu_n)} q$$

$$\tilde{O}_q^{\mu_1\cdots\mu_n} = \overline{q} iD^{(\mu_1} \ldots iD^{\mu_{n-1}}\gamma^{\mu_n)}\gamma^5 q$$

- off-forward matrix elements of the twist two operators

$$\langle P', S'|O_q^{\mu_1\ldots\mu_n}|P,S\rangle = \overline{U}(P',S')[\sum_{\substack{i=0\\ \text{even}}}^{n-1} A_{ni}^q(t) K_{ni}^A(P',P) + \sum_{\substack{i=0\\ \text{even}}}^{n-1} B_{ni}^q(t) K_{ni}^B(P',P) + \delta_{\text{even}}^n C_n^q(t) K_n^C(P',P)]U(P,S)$$

$$\langle P', S'|\tilde{O}_q^{\mu_1\ldots\mu_n}|P,S\rangle = \overline{U}(P',S')[\sum_{\substack{i=0\\ \text{even}}}^{n-1} \tilde{A}_{ni}^q(t) \tilde{K}_{ni}^A(P',P) + \sum_{\substack{i=0\\ \text{even}}}^{n-1} \tilde{B}_{ni}^q(t) \tilde{K}_{ni}^B(P',P)]U(P,S)$$

Basic Properties of Generalized Form Factors

- moments of parton distributions - $\langle P|O_q^{\mu_1\cdots\mu_n}|P\rangle$ and $\langle P|\tilde{O}_q^{\mu_1\cdots\mu_n}|P\rangle$

$$A_{n0}^q(0) = \int_{-1}^{1} dx\; x^{n-1}\, q(x) \quad \text{and} \quad \tilde{A}_{n0}^q(0) = \int_{-1}^{1} dx\; x^{n-1}\, \Delta q(x)$$

- form factors - $\mathcal{O}_q^\mu = \overline{q}\gamma^\mu q$ and $\tilde{\mathcal{O}}_q^\mu = \overline{q}\gamma^\mu\gamma^5 q$

$$A_{10}^q(t) = F_1^q(t) \quad \text{and} \quad B_{10}^q(t) = F_2^q(t)$$

$$\tilde{A}_{10}^q(t) = G_A^q(t) \quad \text{and} \quad \tilde{B}_{10}^q(t) = G_P^q(t)$$

- quark angular momenta [1]

- transverse quark distributions [2]

$$\int_{-1}^{1} dx\; x^{n-1}\, q(x,\vec{b}_\perp) \;=\; \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{n0}^q(-\vec{\Delta}_\perp^2)$$

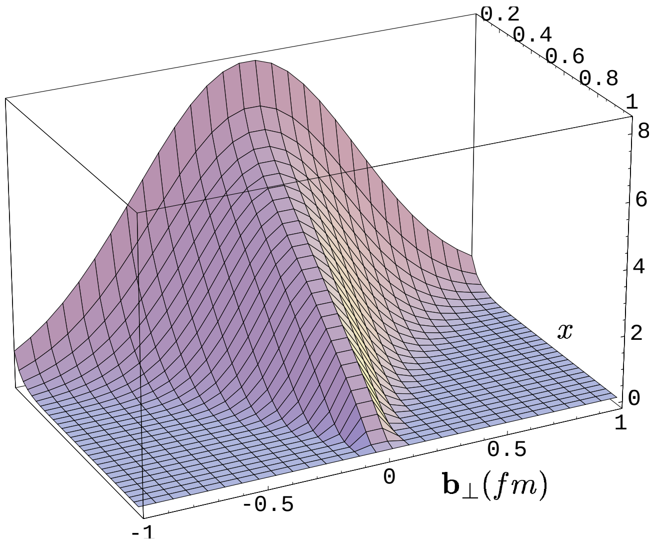
$$\int_{-1}^{1} dx\; x^{n-1}\, \Delta q(x,\vec{b}_\perp) \;=\; \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \tilde{A}_{n0}^q(-\vec{\Delta}_\perp^2)$$

[1] X. D. Ji hep-ph/9603249
[2] M. Burkardt hep-ph/0005108

Full QCD Calculation with Heavy Quarks

- SESAM gauge fields
- Wilson gluons and $N_F = 2$ Wilson fermions
- $a = 0.095$ fm, $L = 1.52$ fm
- $M_\pi = 753(10), 835(13), 895(15)$ MeV

Transverse Distributions



$$A^q_{n0}(-\vec{\Delta}^2_\perp) = \int d^2b_\perp \; e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int_{-1}^{1} dx \; x^{n-1} \, q(x,\vec{b}_\perp)$$

$$\langle b^2_\perp\rangle^q_{(n)} = -4\,\frac{A^{q\,\prime}_{n0}(0)}{A^q_{n0}(0)}$$

- at $x = 1$ a single quark carries all the momentum

$$\lim_{x\to 1} q(x,\vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

- higher moments A^q_{n0} weight $x \sim 1$ more heavily

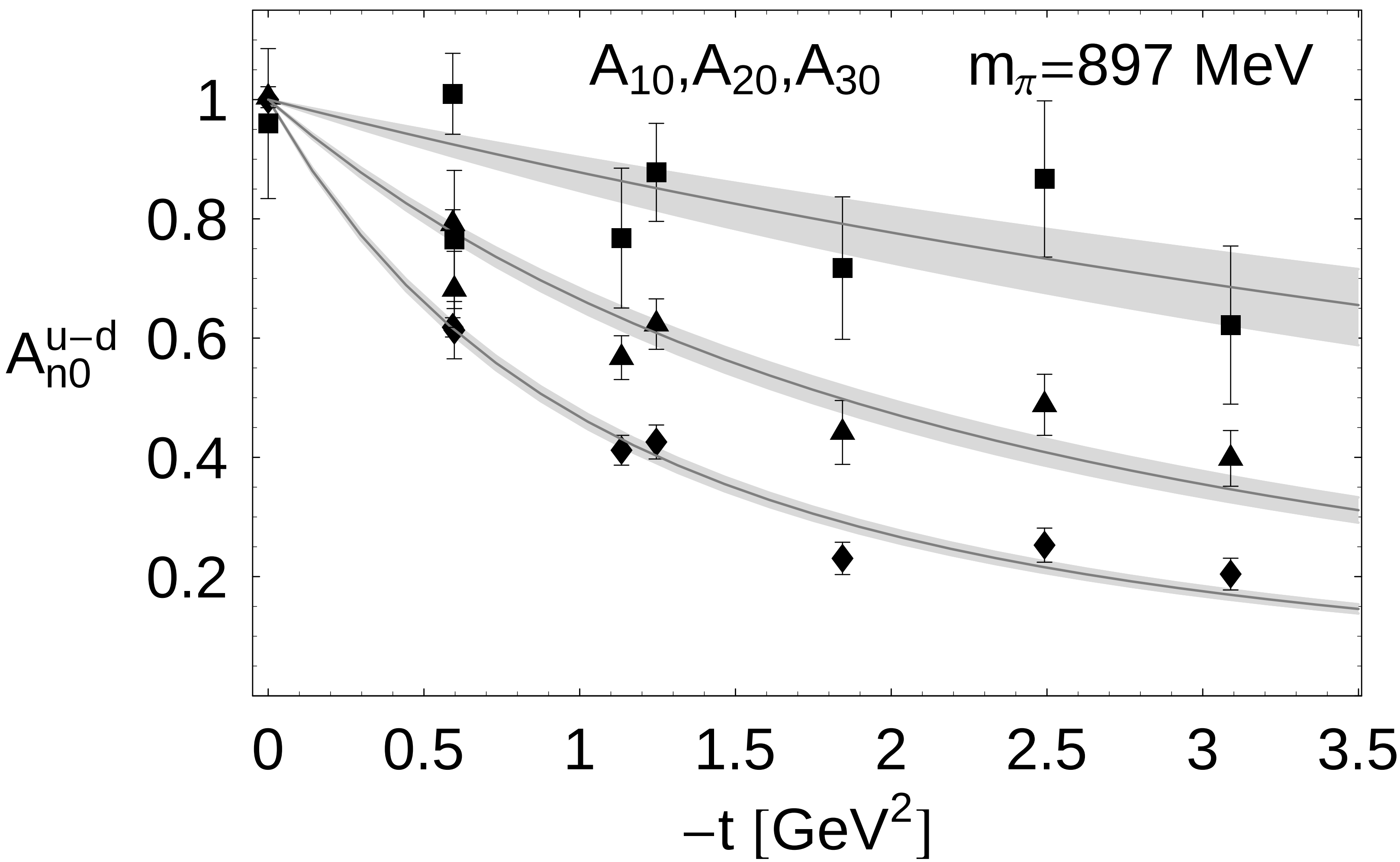
$$\lim_{n\to\infty} A^q_{n0}(t) \propto \int d^2b_\perp \; e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\delta^2(\vec{b}_\perp) = \text{constant}$$

- slopes of A^q_{n0} should decrease as n increases

- A_{10}, A_{30}, $\tilde{A}_{20}$ measure $q - \overline{q}$ & $\tilde{A}_{10}$, $\tilde{A}_{30}$, A_{20} measure $q + \overline{q}$

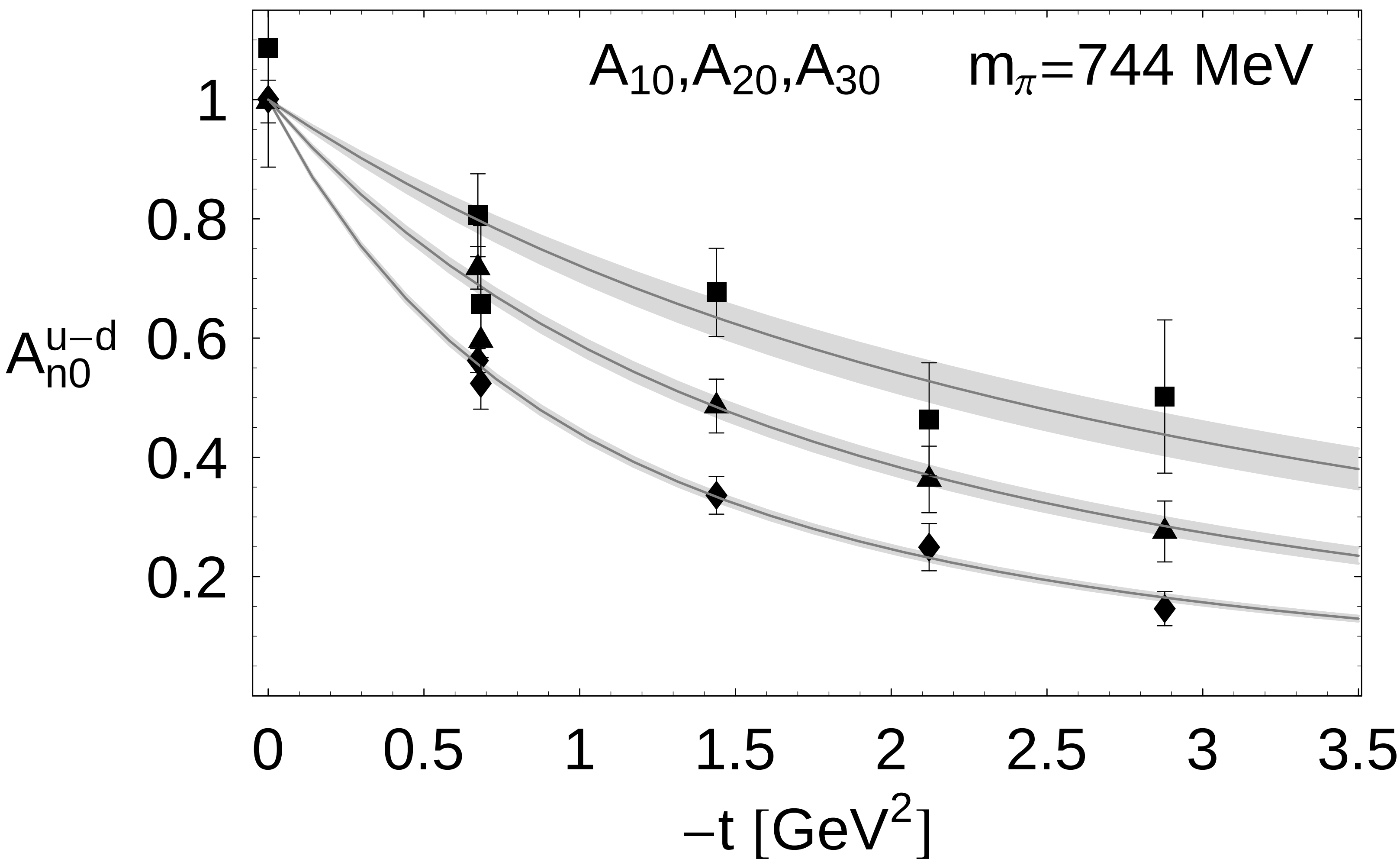
Transverse Distributions: $m_\pi = 897$ MeV

- slope of $A_{10}^{u-d} = -0.93 \pm 0.04\ (\mathrm{GeV})^{-2}$
- slope of $A_{30}^{u-d} = -0.13 \pm 0.03\ (\mathrm{GeV})^{-2}$ (factor of 7)



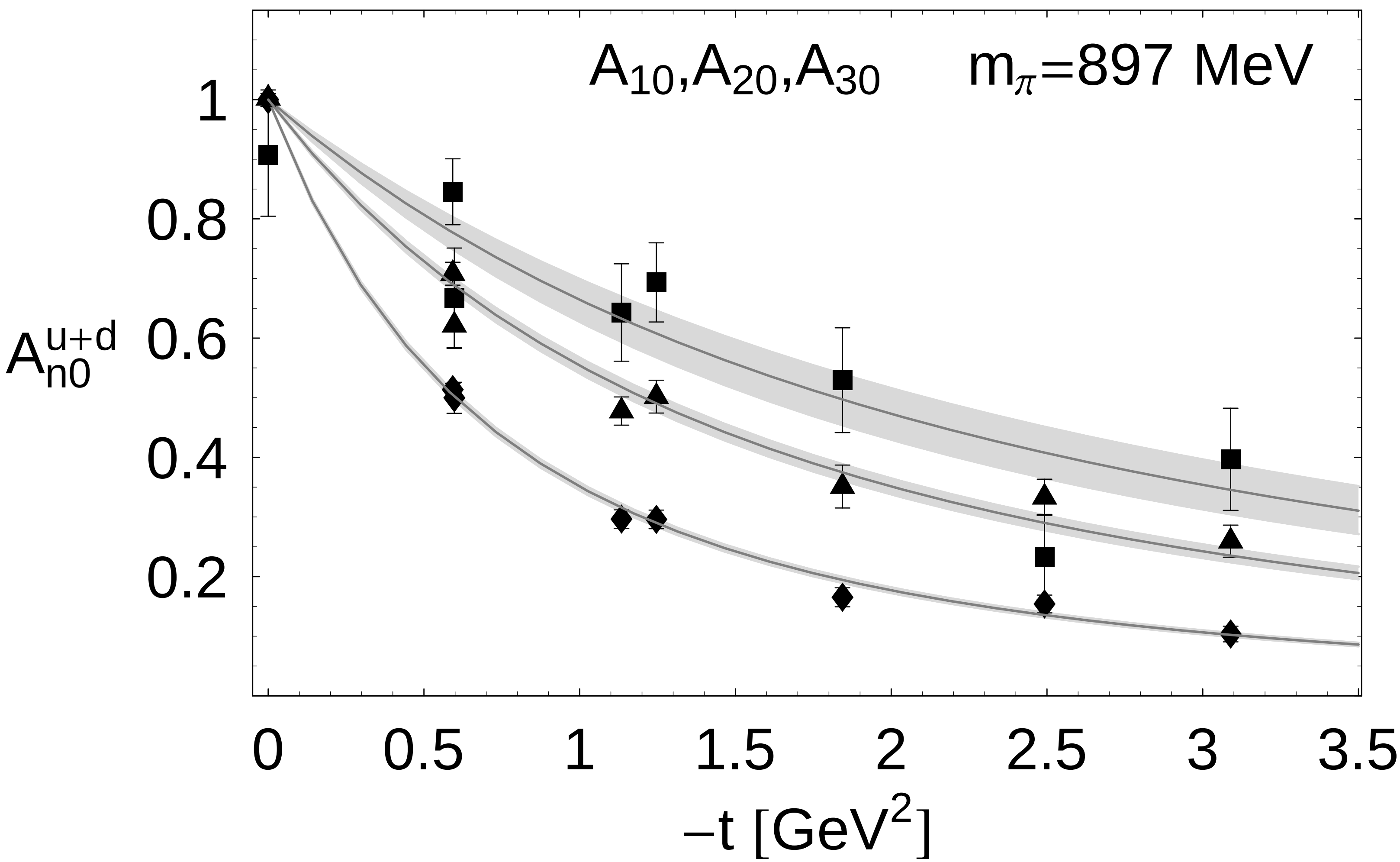
Transverse Distributions: Mass Dependence

- slope of $A_{10}^{u-d} = -1.02 \pm 0.03\ (\mathrm{GeV})^{-2}$
- slope of $A_{30}^{u-d} = -0.36 \pm 0.04\ (\mathrm{GeV})^{-2}$ (factor of 3)



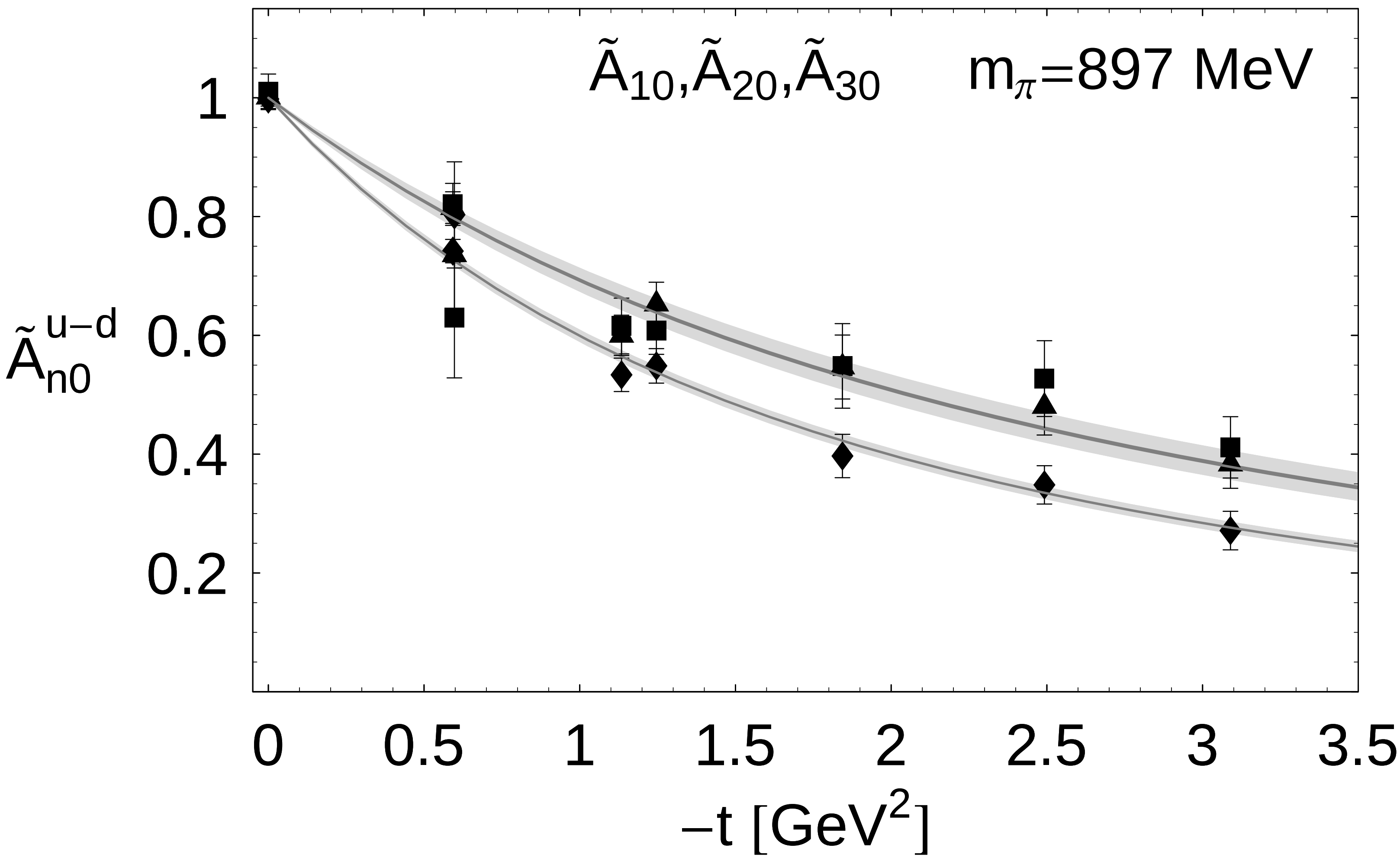
Transverse Distributions: Flavor Dependence

- slope of $A_{10}^{u+d} = -1.38 \pm 0.03\ (\mathrm{GeV})^{-2}$
- slope of $A_{30}^{u+d} = -0.45 \pm 0.07\ (\mathrm{GeV})^{-2}$ (factor of 3)



Transverse Distributions: Spin Dependence

- slope of $\tilde{A}_{10}^{u-d} = -0.58 \pm 0.02\ (\text{GeV})^{-2}$
- slope of $\tilde{A}_{30}^{u-d} = -0.40 \pm 0.03\ (\text{GeV})^{-2}$ (factor of 1.5)

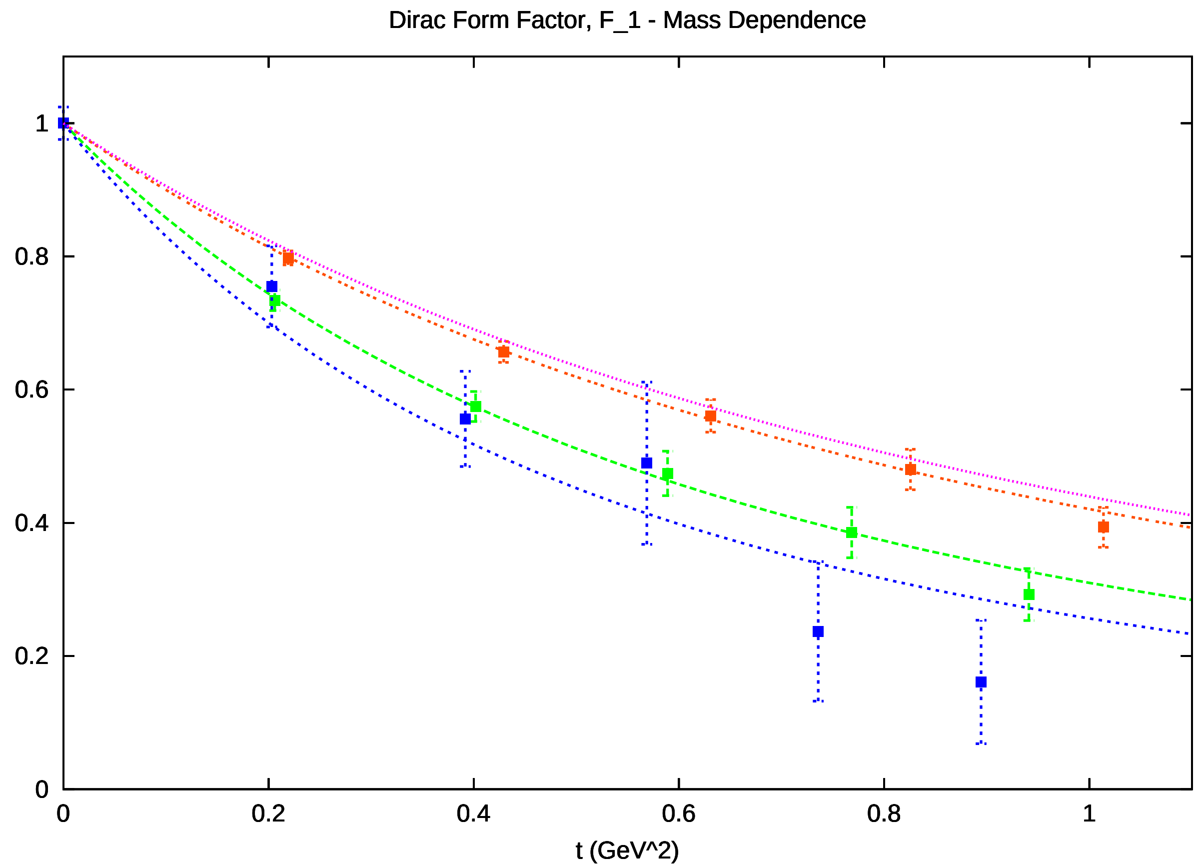


Hybrid Calculation with Lighter Quarks

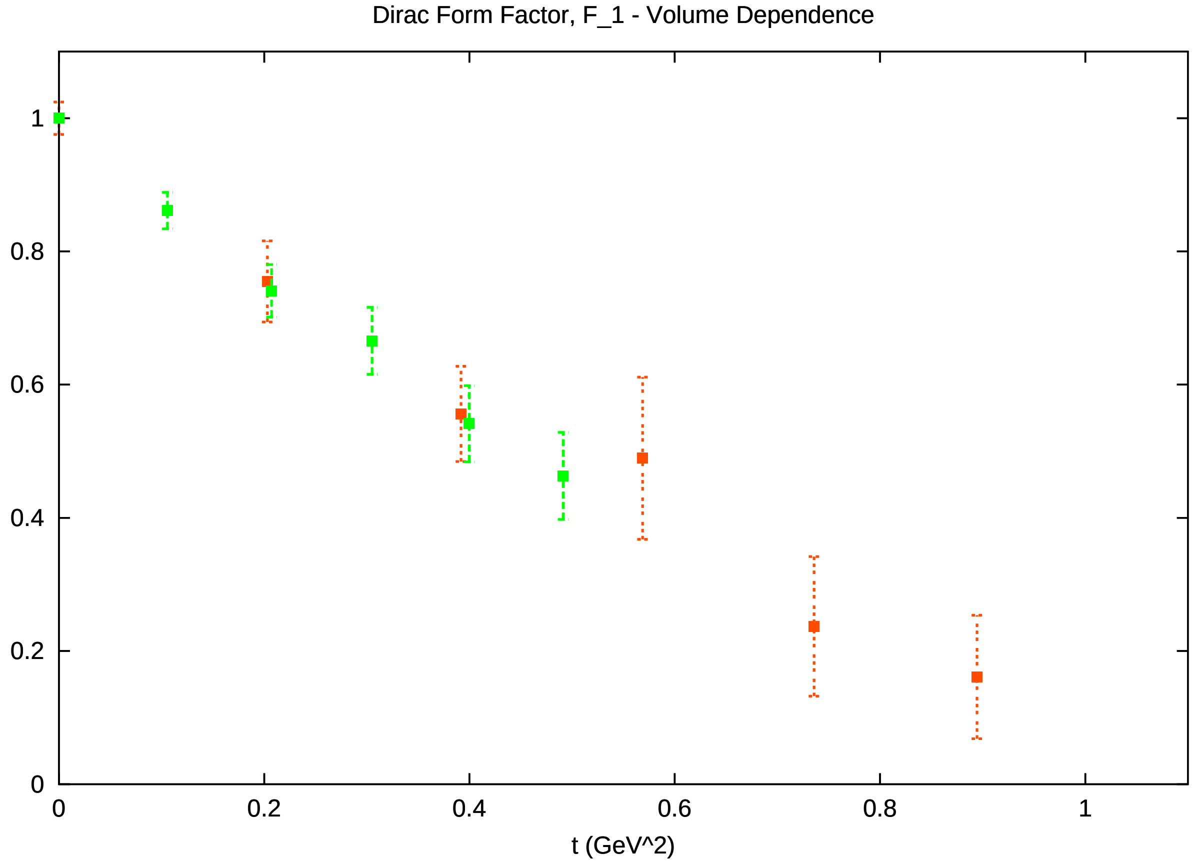
- asqtad staggered sea quarks (MILC)
- domain wall valence quarks - $M = 1.7$ with HYP Smearing - $\alpha_1 = 0.75$, $\alpha_2 = 0.6$, $\alpha_3 = 0.3$

$am_{u/d}^{\text{asqtad}}$	L/a	a	L	m_π^{asqtad}	$am_{u/d}^{\text{DWF}}$	m_π^{DWF}	#
		fm	fm	MeV		MeV	
0.05	20	0.131	2.62	730(3)	0.0810	725(4)	107
0.03	20	0.132	2.64	564(2)	0.0478	570(3)	134
0.01	20	0.135	2.70	329(1)	0.0138	337(5)	104
0.01	28		3.78		0.0138	333(2)	131

F_1 Form Factor - Mass Dependence

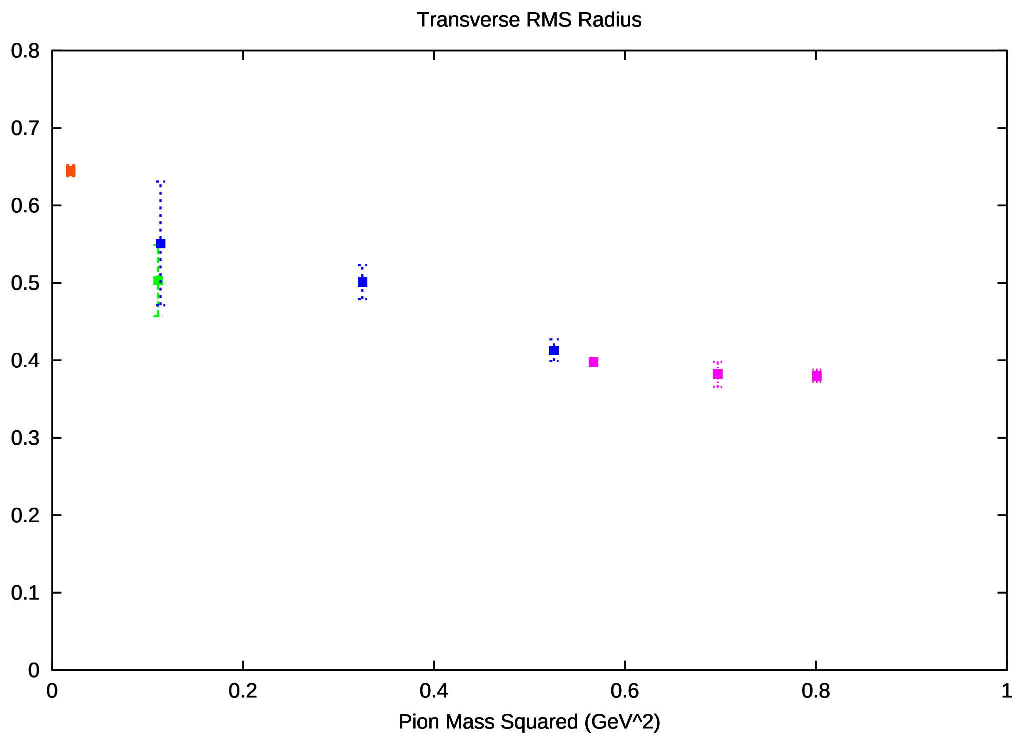


F_1 Form Factor - Volume Dependence



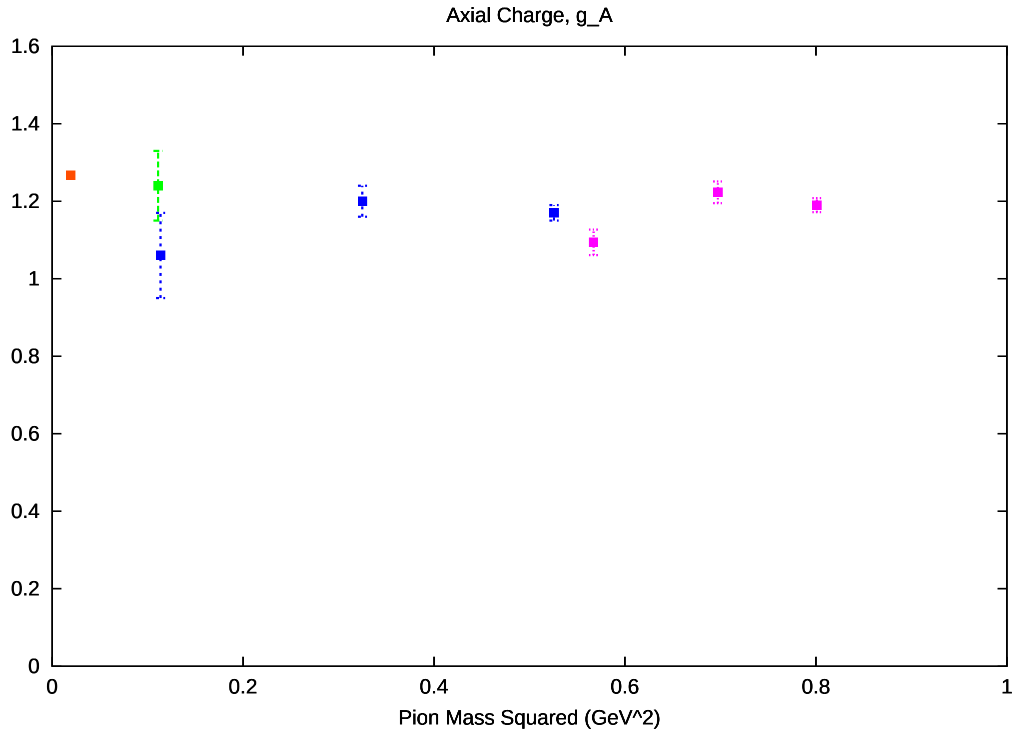
Transverse RMS Radius $\sqrt{r_\perp^2}$

m_π^{DWF}	L/a	Z_V	$\sqrt{r_\perp^2}$ (fm)
725	20	1.153(2)	0.413(14)
570	20	1.132(3)	0.501(22)
337	20	1.117(27)	0.551(80)
333	28	1.108(7)	0.503(46)



Axial Charge g_A

m_π^{DWF}	L/a	Z_A	Z_V	g_A
725	20	1.1282( )	1.153(2)	1.17(2)
570	20	1.1066(6)	1.132(3)	1.20(4)
337	20	1.0852(8)	1.117(27)	1.06(11)
333	28	1.0838(1)	1.108(7)	1.24(9)



Conclusions

- the transverse size of the nucleon $\langle r_{\perp}^2 \rangle$, in the heavy pion world, shows a significant dependence on the longitudinal momentum $\langle x \rangle$

- g_A and F_1 patch smoothly between the SESAM and MILC lattices and approach the experimental results in the lighter pion world

- continuing with the complete calculation of the generalized form factors with additional quark masses and extended statistics