Transverse Structure of Nucleon Parton Distributions

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February 23, 2004

Generalized parton distributions determine the angular momentum decomposition of the nucleon and the transverse distribution of partons in the nucleon. Additionally, in particular limits they reduce to form factors and ordinary parton distributions. I will review generalized parton distributions and present our full QCD lattice calculations of the quark helicity and orbital angular momentum in the nucleon, the asymptotic scaling ratio F_2/F_1, and the transverse distribution of quarks in the nucleon.

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http://talks.drubryantrenner.org/jlab_2-23-04.pdf

In Short

Generalized Parton Distributions determine

- 3D distribution of quarks in a mixed representation - 2 transverse coordinates and 1 longitudinal momentum

- decomposition of nucleon spin into quark helicity, quark orbital, and gluon contributions

- form factors and ordinary parton distributions

Outline

I. Continuum Physics Review

II. Transverse Nucleon Structure

III. Lattice Calculation

IV. Our Results

Continuum Physics Review

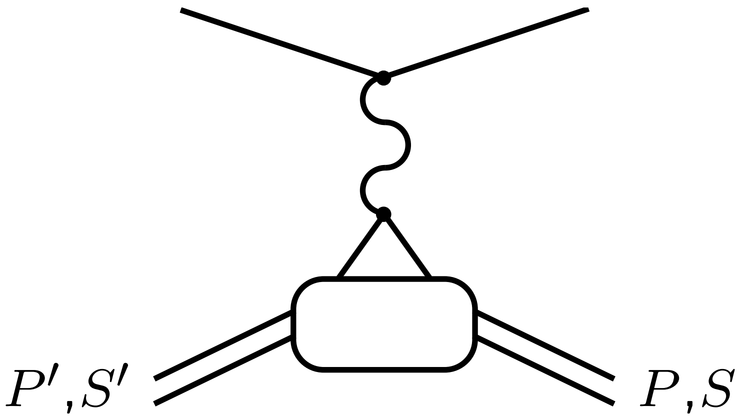
I. Form Factors

II. Parton Distributions

III. Generalized Parton Distributions

Form Factors

- lepton-nucleon scattering, $lN \to lN$, elastic



$$\Delta = P' - P$$
$$t = \Delta^2$$

- off-forward matrix element of the electromagnetic current

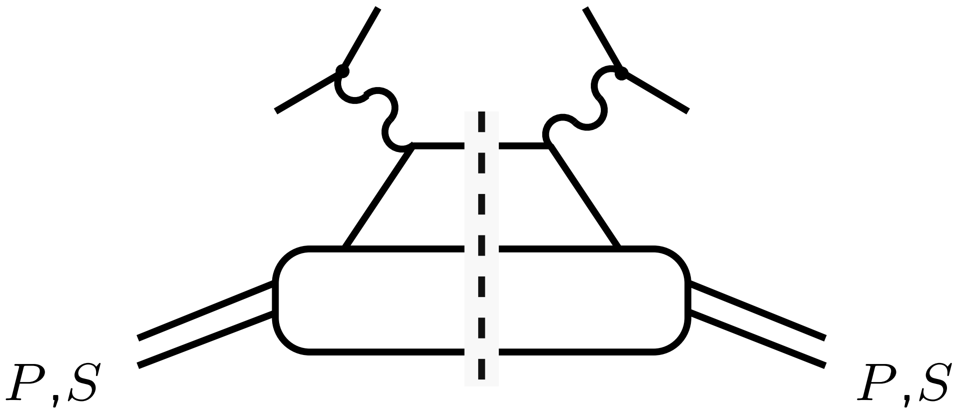
$$< P', S' | J^\mu | P, S >= \overline{U}(P', S') \left(\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} F_2(t) \right) U(P, S)$$

- interpretation as Fourier transform of charge and current densities *in certain cases*

- magnetic moment, charge & current radii

Parton Distributions

- deep inelastic scattering, $lN \to lX$, inclusive



$$P^+ = \frac{1}{2}\left(P^t + P^z\right)$$
$$y^- = \frac{1}{2}\left(y^t - y^z\right)$$

- forward matrix element of light-cone quark correlator

$$\mathcal{O}_q(x, \vec{b}_\perp) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-}\, \bar{q}\left(-\frac{y^-}{2}, \vec{b}_\perp\right) \gamma^+ q\left(\frac{y^-}{2}, \vec{b}_\perp\right)$$

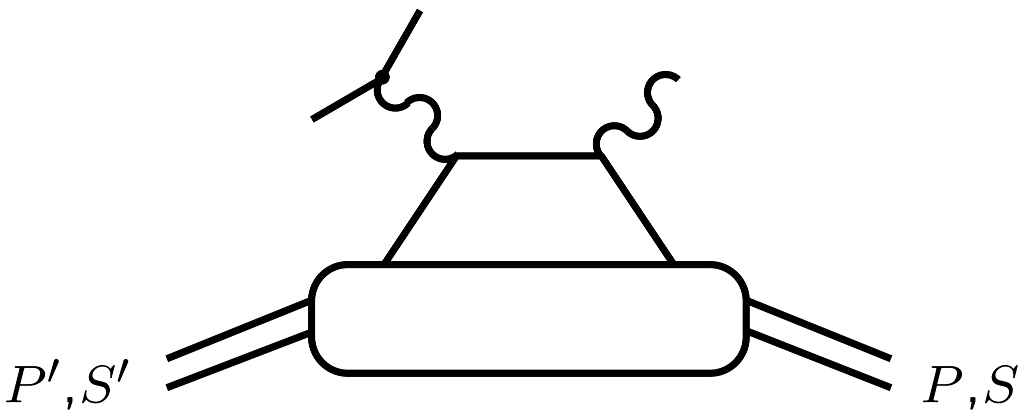
$$\left\langle P, S \middle| \mathcal{O}_q(x, \vec{0}_\perp) \middle| P, S \right\rangle = q(x)$$

- longitudinal momentum distribution in the *infinite momentum frame*

- momentum fraction & spin fraction

Generalized Parton Distributions

- deeply virtual Compton scattering, $lN \to lN\gamma$, exclusive



$$\overline{P} = \tfrac{1}{2}\,(P' + P)$$

$$\xi = -\frac{\Delta^+}{2\overline{P}^+}$$

- off-forward matrix element of light-cone quark correlator

$$\left\langle P', S' \left| \mathcal{O}_q(x, \vec{0}_\perp) \right| P, S \right\rangle =$$

$$\frac{1}{2\overline{P}^+}\overline{U}(P', S')\left(\gamma^+ H(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M}E(x,\xi,t)\right)U(P,S)$$

- meson production, Compton scatering

GPD reviews: Ji hep-ph/9807358, Diehl hep-ph/0307382

Generalized Parton Distributions

- familiar limits: parton distributions & form factors

$$H_q(x,0,0) = q(x)$$

$$\sum_q e_q \int dx\ H_q(x,\xi,t) = F_1(t)$$

$$\sum_q e_q \int dx\ E_q(x,\xi,t) = F_2(t)$$

- quark angular momentum [1]

$$J_q = \frac{1}{2}\int dx\ x\,(H_q(x,0,0) + E_q(x,0,0))$$

- transverse quark distribution in *infinite momentum frame* [2]

$$\int d^2b_\perp\ e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} q(x,\vec{b}_\perp) = H_q(x,0,-\vec{\Delta}_\perp^2)$$

[1] Ji hep-ph/9603249
[2] Burkardt hep-ph/0005108

Transverse Structure

I. Charge Distribution: Non-Relativistic Limit

II. Transverse Distribution: Infinite Momentum Frame

Charge Distribution

- wave packet

$$|\psi\rangle = \int \frac{d^3p}{(2\pi)^{3/2}} \frac{\psi(\vec{p})}{\sqrt{2E_{\vec{p}}}} |\vec{p}\rangle$$

- Fourier transform of charge distribution

$$\int d^3x \; e^{i\vec{q}\cdot\vec{x}} \; \langle\psi| \, J^0(\vec{x}) \, |\psi\rangle =$$

$$= \int d^3p \; \psi^*(\vec{k}) \; \psi(\vec{p}) \; < \vec{k}|J^0(0)|\vec{p} > / \left(2\sqrt{E_{\vec{k}}E_{\vec{p}}}\right)$$

$$= \int d^3p \; \psi^*(\vec{k}) \; \psi(\vec{p}) \; F(q^2) \; \left(E_{\vec{k}} + E_{\vec{p}}\right) / \left(2\sqrt{E_{\vec{k}}E_{\vec{p}}}\right)$$

where $\vec{k} = \vec{p} + \vec{q}$ & $< \vec{k}|J^0(0)|\vec{p} >= F(q^2) \left(E_{\vec{k}} + E_{\vec{p}}\right)$

Burkardt hep-ph/0005108

Charge Distribution

- non-relativistic limit $\quad |\vec{q}| \ll M \quad E_{\vec{k}} \approx E_{\vec{p}} \quad q^2 \approx -\vec{q}^{\,2}$
- broad wave packet $\quad \psi(\vec{k}) \approx \psi(\vec{p})$

$$
\begin{aligned}
&\int d^3x \; e^{i\vec{q}\cdot\vec{x}} \, \langle\psi|\, J^0(\vec{x})\, |\psi\rangle \\
&= \int d^3p \; \psi^*(\vec{k}) \; \psi(\vec{p}) \; F(q^2) \; \left(\frac{E_{\vec{k}} + E_{\vec{p}}}{2\sqrt{E_{\vec{k}} E_{\vec{p}}}}\right) \\
&\approx \int d^3p \; \psi^*(\vec{p}) \; \psi(\vec{p}) \; F(-\vec{q}^{\,2}) \\
&\approx F(-\vec{q}^{\,2})
\end{aligned}
$$

Charge Distribution

- separation of scales, $L_{FF/WP}$ is the form factor/wave packet length scale

$$L_{FF} \sim \frac{1}{|\vec{q}|} \gg L_{WP} \gg \frac{1}{M} \qquad \Rightarrow \qquad L_{FF} \gg \frac{1}{M}$$

- hydrogen atom

$$L_{FF} = \frac{1}{\alpha m_e} = 7 \cdot 10^4 \text{ fm} \quad \gg \quad \frac{1}{M_H} = \frac{1}{M_P} = 0.2 \text{ fm}$$

- proton

$$L_{FF} = 0.87 \text{ fm} \quad \approx \quad \frac{1}{M_P} = 0.2 \text{ fm}$$

Belitsky, Ji, Yuan hep-ph/0307383

Transverse Distribution

- wave packet

$$|\psi\rangle = \int \frac{d^2p_\perp}{(2\pi)} \frac{\psi(\vec{p}_\perp)}{\sqrt{2E_{\vec{p}}}} |\vec{p}_\perp, P_z\rangle \qquad \vec{p} = (\vec{p}_\perp, P_z)$$

- Fourier transform of transverse quark distribution

$$\mathcal{O}_q(x, \vec{b}_\perp) = \int \frac{dy^-}{4\pi} e^{ix\overline{P}^+y^-} \bar{q}\left(-y^-/2, \vec{b}_\perp\right) \gamma^+ q\left(y^-/2, \vec{b}_\perp\right)$$

$$\int d^2b_\perp \, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \langle\psi| \mathcal{O}_q(x, \vec{b}_\perp) |\psi\rangle =$$

$$\int d^2p_\perp \, \psi^*(\vec{k}_\perp) \, \psi(\vec{p}_\perp) \, H_q(x, 0, \Delta^2) \left(2\sqrt{E_{\vec{k}}}\sqrt{E_{\vec{p}}}\right)$$

$$\vec{k}_\perp = \vec{p}_\perp + \vec{q}_\perp \qquad k_z = p_z = P_z \qquad < \vec{k}|\mathcal{O}_q(x, \vec{0}_\perp)|\vec{p} >= H_q(x, 0, \Delta^2)$$

Transverse Distribution

- infinite momentum limit $\quad |\vec{q}_\perp| \ll P_z \quad E_{\vec{k}} \approx E_{\vec{p}} \quad \Delta^2 \approx -\vec{\Delta}_\perp^2$

- broad wave packet $\quad \psi(\vec{k}_\perp) \approx \psi(\vec{p}_\perp)$

$$
\begin{aligned}
&\int d^2 b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \langle \psi | \, \mathcal{O}_q(x, \vec{b}_\perp) \, | \psi \rangle \\
&= \int d^2 p_\perp \; \psi^*(\vec{k}_\perp) \; \psi(\vec{p}_\perp) \; H_q(x, 0, \Delta^2) \, / \left(2\sqrt{E_{\vec{k}}}\sqrt{E_{\vec{p}}} \right) \\
&\approx \int d^2 p_\perp \; \psi^*(\vec{p}_\perp) \; \psi(\vec{p}_\perp) \; H_q(x, 0, -\vec{\Delta}_\perp^2)/2P_z \\
&\approx H_q(x, 0, -\vec{\Delta}_\perp^2)
\end{aligned}
$$

Transverse Distribution

- separation of scales, $L_{TD/WP}$ is the transverse distribution/wave packet length scale

$$L_{TD} \sim \frac{1}{\left|\vec{\boldsymbol{\Delta}}_\perp\right|} \gg L_{WP} \gg \frac{1}{\sqrt{M^2+P_z^2}} \qquad \Rightarrow \qquad L_{TD} \gg \frac{1}{\sqrt{M^2+P_z^2}} \to 0$$

- transverse quark distribution

$$\int d^2b_\perp \; e^{i\vec{\boldsymbol{\Delta}}_\perp\cdot\vec{b}_\perp} q(x,\vec{b}_\perp) = H_q(x,0,-\vec{\boldsymbol{\Delta}}_\perp^2)$$

Lattice Calculation

I. Moments of Generalized Parton Distributions

II. Generalized Form Factors

III. Matrix Elements from Lattice QCD

Moments of Generalized Parton Distributions

- generalized parton distribtuions

$$\left\langle P',S'\left|\mathcal{O}_q(x)\right|P,S\right\rangle \;=\; \frac{1}{2\overline{P}^+}\overline{U}(P',S')\left(\gamma^+ H_q(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M}E_q(x,\xi,t)\right)U(P,S)$$

$$\mathcal{O}_q(x,\vec{b}_\perp) = \int\frac{dy^-}{4\pi}\,e^{ixP^+y^-}\,\overline{q}\left(-\frac{y^-}{2},\vec{b}_\perp\right)\gamma^+\,q\left(\frac{y^-}{2},\vec{b}_\perp\right)$$

- light-cone expansion & tower of twist 2 operators

$$\mathcal{O}_q^{\mu_1\mu_2\ldots\mu_n} = \overline{q}\gamma^{\{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}q$$

- moments of generalized parton distribtuions

$$\left\langle P'|\mathcal{O}^{\mu_1\mu_2\ldots\mu_n}|P\right\rangle \sim \int dx\,x^{n-1}H_q(x,\xi,t) \quad \& \quad \int dx\,x^{n-1}E_q(x,\xi,t)$$

Generalized Form Factors

- $n + 1$ generalized form factors

$$\left\langle P' | \mathcal{O}_q^{\mu_1 \mu_2 \cdots \mu_n} | P \right\rangle \sim A_{ni}^q(t),\ B_{ni}^q(t),\ C_n^q(t)$$

- moments of parton distributions

$$\left\langle x^{n-1} \right\rangle_q = A_{n0}^q(0)$$

- form factors

$$F_1(t) = \sum_q e_q A_{10}^q(t) \qquad F_2(t) = \sum_q e_q B_{10}^q(t)$$

- quark angular momentum

$$J_q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0) \right)$$

Ji, Lebed hep-ph/0012160

Generalized Form Factors

- moments of generalized parton distributions

$$\int_{-1}^{1} dx\; x^{n-1} H_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} A^q_{n,2i}(t)(-2\xi)^{2i} + \text{mod}(n+1,2)C^q_n(t)(-2\xi)^n$$

$$\int_{-1}^{1} dx\; x^{n-1} E_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} B^q_{n,2i}(t)(-2\xi)^{2i} - \text{mod}(n+1,2)C^q_n(t)(-2\xi)^n$$

- transverse momentum transfer, $\xi \to 0$

$$\int_{-1}^{1} dx\; x^{n-1} H_q(x,0,t) = A^q_{n0}(t)$$

$$\int_{-1}^{1} dx\; x^{n-1} E_q(x,0,t) = B^q_{n0}(t)$$

Lattice Correlation Functions

- correlation function of $2N$ fermion fields $(\psi,\overline{\psi})$ and M gauge links $(U = e^{iagA})$

$$\left\langle \psi_1 \cdots \psi_N \, \overline{\psi}_1 \cdots \overline{\psi}_N \, U_1 \cdots U_M \right\rangle =$$

$$= \int DU \int D\psi D\overline{\psi} \; e^{-S_G[U]} e^{-\overline{\psi} M[U] \psi} \psi_1 \cdots \psi_N \, \overline{\psi}_1 \cdots \overline{\psi}_N \, U_1 \cdots U_M$$

$$= \int DU \; e^{-S_G[U]} \, \mathrm{det}^{N_f}(M[U]) \; U_1 \cdots U_M \; \sum_\pi (-1)^\pi M[U]^{-1}_{1\pi_1} \cdots M[U]^{-1}_{N\pi_N}$$

- numerical integration

$$\int DU \; e^{-S_G[U]} \, \mathrm{det}^{N_f}(M[U]) \; F[U] = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} F[U^i]$$

Matrix Elements from Lattice Correlation Functions

- nucleon interpolating field with gauge invariant smearing

$$J^{\alpha} = U_a^{\alpha}(U_b^T C\gamma^5 D_c)\epsilon_{abc}$$

- lattice ratios: fixed source (src), momentum projection at operator (op), sink (snk)

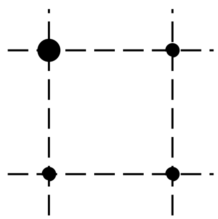
$$\left\langle p', s' | \mathcal{O} | p, s \right\rangle \sim \frac{\left\langle J(x_{\mathsf{snk}})\ \mathcal{O}(x_{\mathsf{op}})\ \overline{J}(x_{\mathsf{src}}) \right\rangle}{\left\langle J(x_{\mathsf{snk}})\ \overline{J}(x_{\mathsf{src}}) \right\rangle}$$

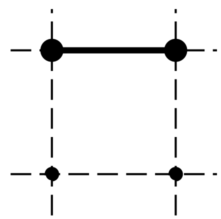
- quantum mechanical matrix elements are given by appropriate ratios of lattice correlation fuctions

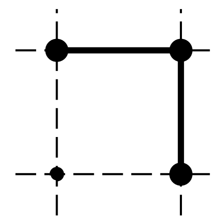
Building Blocks

- two quark operators can be written in terms of a basic set of building blocks

$\overline{q}(x)\,\Gamma_i\,q(x)$ $\quad$ $\overline{q}(x+\widehat{\mu})\,\Gamma_i\,U^\dagger_\mu(x)\,q(x)$ $\quad$ $\overline{q}(x+\widehat{\nu}+\widehat{\mu})\,\Gamma_i\,U^\dagger_\nu(x+\widehat{\mu})U^\dagger_\mu(x)\,q(x)$







- Γ_i $(i=1,\ldots,16)$ denote a complete basis of 4×4 Dirac spin matrices

- calculate upto 3 gauge link insertions: $U^\dagger_{\mu_n}(x+\widehat{\mu}_{n-1}+\cdots\widehat{\mu}_1)\cdots U^\dagger_{\mu_1}(x)$

- calculate a broad range of momentum transfers $\vec{q}$

LHPC, SESAM hep-lat/0201021

Overdetermined Set of Lattice Observables

- i labels all combinations of operator indices $(\mu_1, \ldots, \mu_n)$ and momenta P', P

$$\mathcal{O}_i^{\overline{\mathsf{MS}}} = \left\langle P' | \mathcal{O}^{\mu_1 \ldots \mu_n} | P \right\rangle$$

- expand $\mathcal{O}_i^{\overline{\mathsf{MS}}}$ in terms of generalized form factors

$$\mathcal{O}_i^{\overline{\mathsf{MS}}} = \sum_{\alpha=1}^{n+1} A_{i\alpha} F_\alpha(t)$$

- match $\mathcal{O}_i^{\overline{\mathsf{MS}}}$ to $\mathcal{O}_i^{\mathsf{lat}}$ with one loop matching coefficients Z_{ij}

$$\mathcal{O}_i^{\overline{\mathsf{MS}}} = \sum_j Z_{ij} \mathcal{O}_i^{\mathsf{lat}}$$

- construct the appropriate ratio of lattice correlation functions R_i

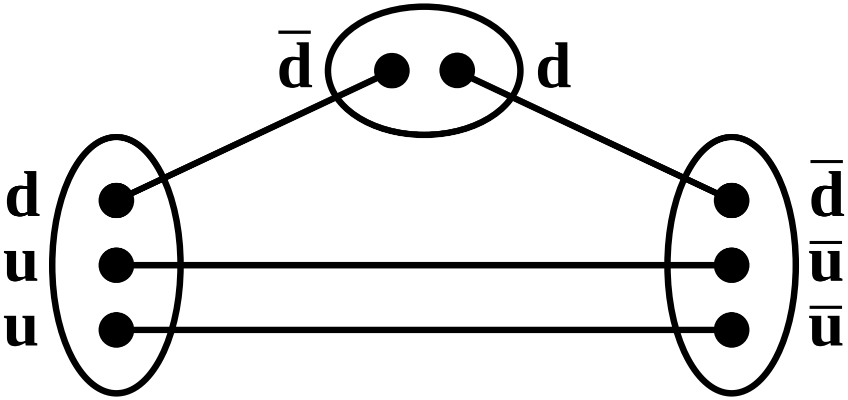
$$R_i = \sum_j B_{ij} \mathcal{O}_j^{\mathsf{lat}}$$

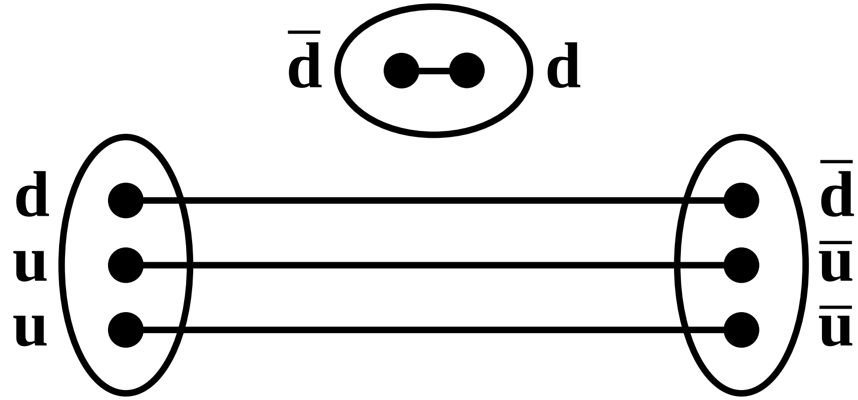
- overdetermined set of observables

$$R_i = \sum_{\alpha=1}^{n+1} C_{i\alpha} F_\alpha(t) \qquad \chi^2 = \sum_i \left(\frac{\sum_{\alpha=1}^{n+1} C_{i\alpha} F_\alpha(t) - R_i}{\sigma_i} \right)^2$$

LHPC, SESAM hep-lat/0304018

Connected & Disconnected Diagrams





- connected diagrams require $18 = 3 \cdot 2 \cdot 3$ Dirac matrix inversions per gauge field, determines non-singlet matrix elements $(u - d)$ for $m_u = m_d$

- disconnected diagrams require $\# \cdot (t_{\text{sink}} - t_{\text{source}}) \cdot V$ Dirac matrix inversions per gauge field, necessary for singlet matrix elements $(u + d)$

- other methods: truncated eigenmode approach, stochastic estimation, wall source technique

Our Results

I. Simulation Parameters

II. Form Factors

III. Quark Angular Momenta

IV. Transverse Quark Distributions

Simulation Parameters

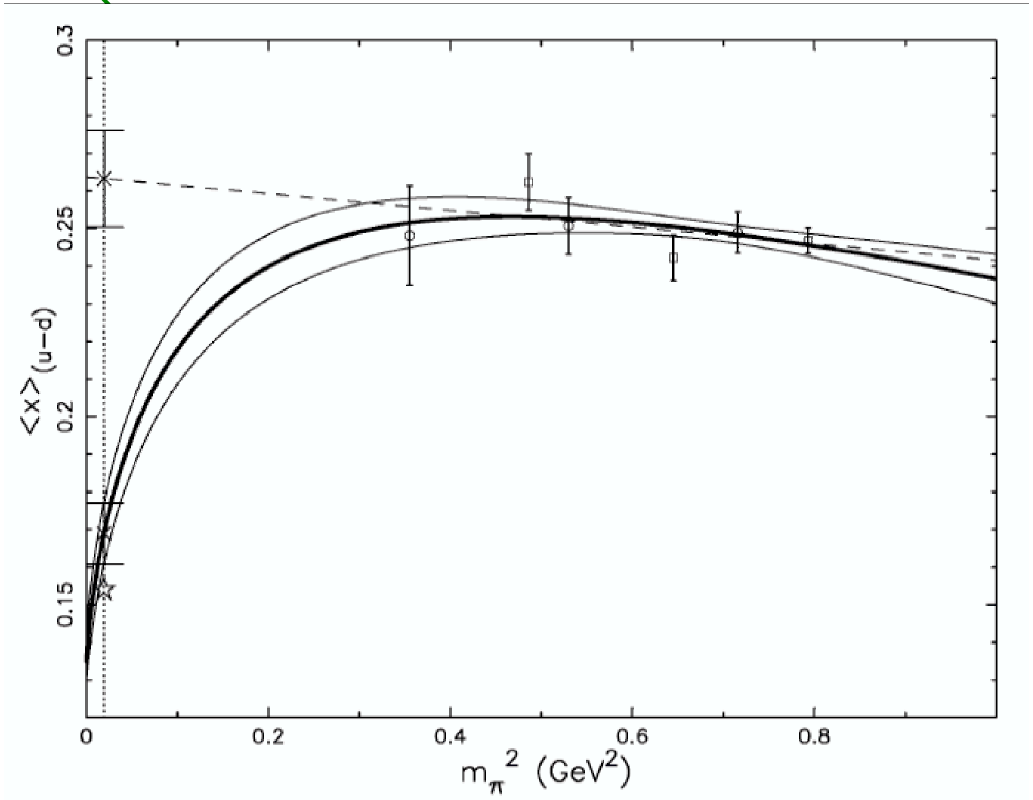
- SESAM gauge fields
- $N_F = 2$ Wilson fermions
- $a = 0.092$ fm, $L = 1.48$ fm, $16^3 \times 32$
- O(200) gluon configurations
- $M_\pi = 744,\ 831,\ 897$ MeV

Chiral Extrapolations

- leading order χPT & heavy quark limit

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_A^2+1)m_\pi^2}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2} \right) \right) + b_n m_\pi^2$$

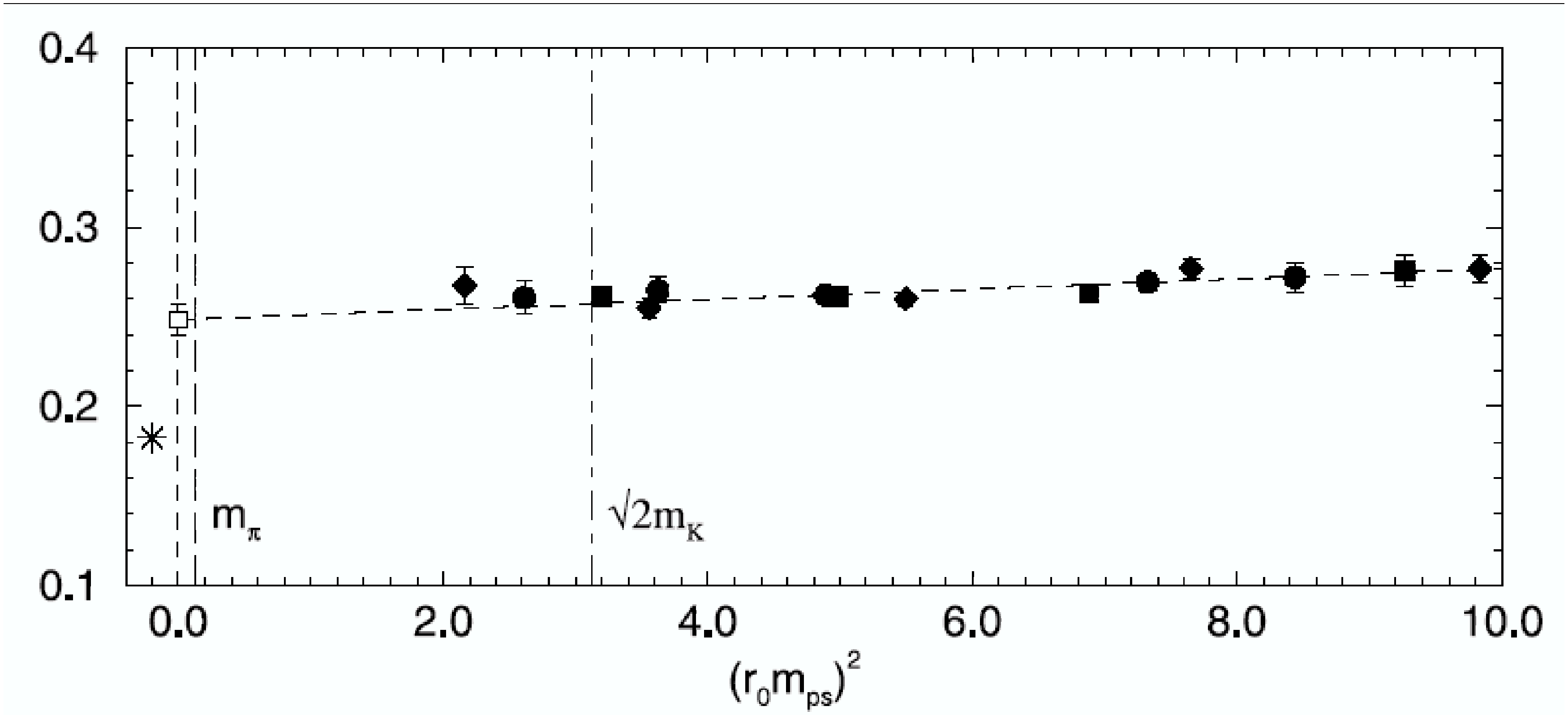
- $\langle x \rangle$ for SESAM full QCD



Detmold, Melnitchouk, Negele, Renner, Thomas hep-lat/0103006

Heavy Pion World

- linear m_π^2 regime
- $\langle x \rangle$ for QCDSF quenched QCD

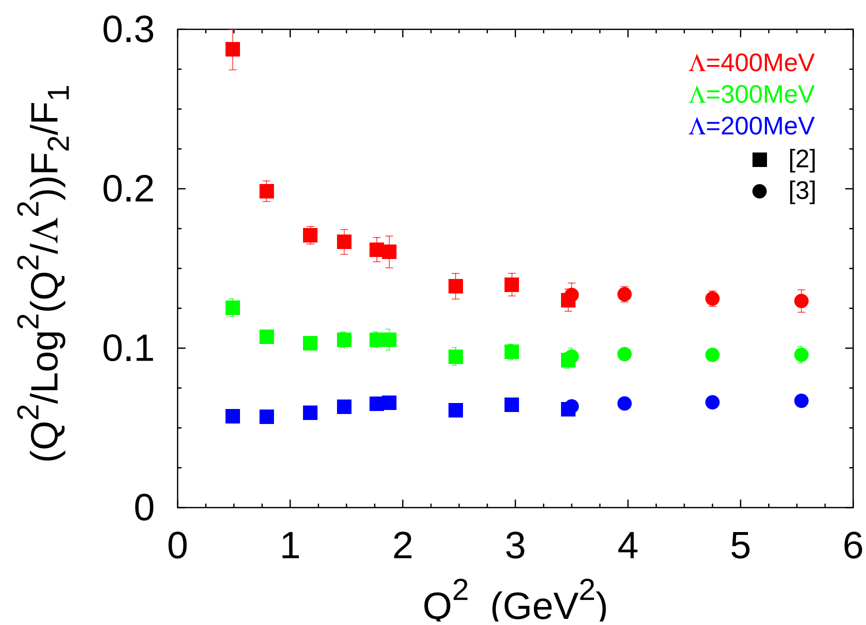


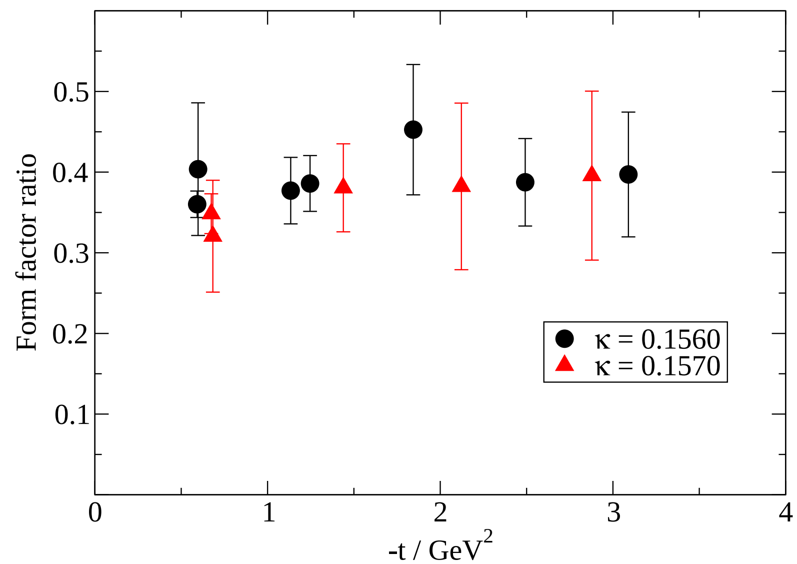
QCDSF hep-lat/0209160

Form Factor Ratios

- perturbative QCD & quark counting rules

$$\frac{Q^2 F_2(Q^2)}{\log^2(Q^2/\Lambda^2) F_1(Q^2)} \sim \text{constant}$$



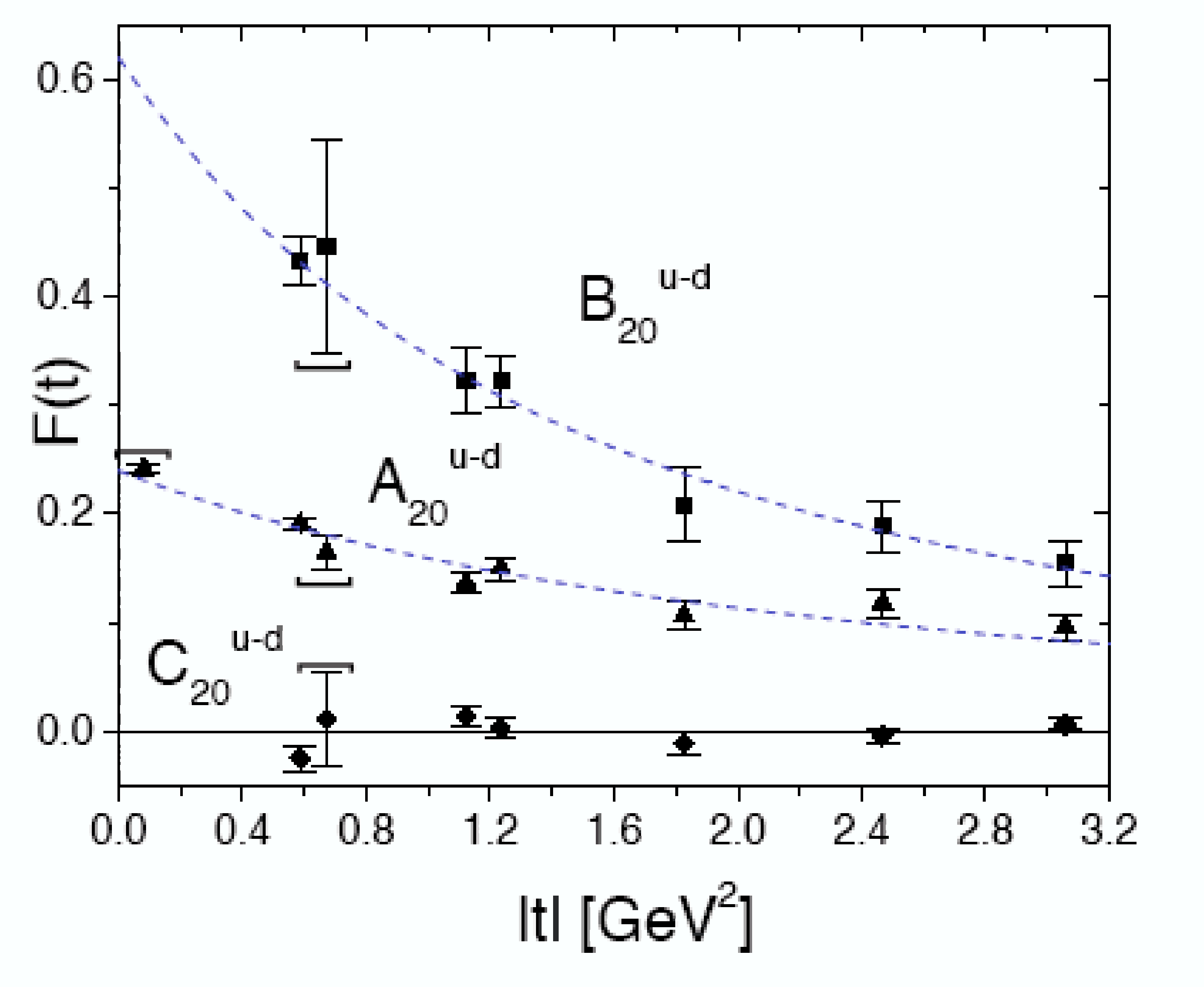


- note normalization difference: compare right plot with 2.79 times left plot

Belitsky, Ji, Yuan hep-ph/0212351

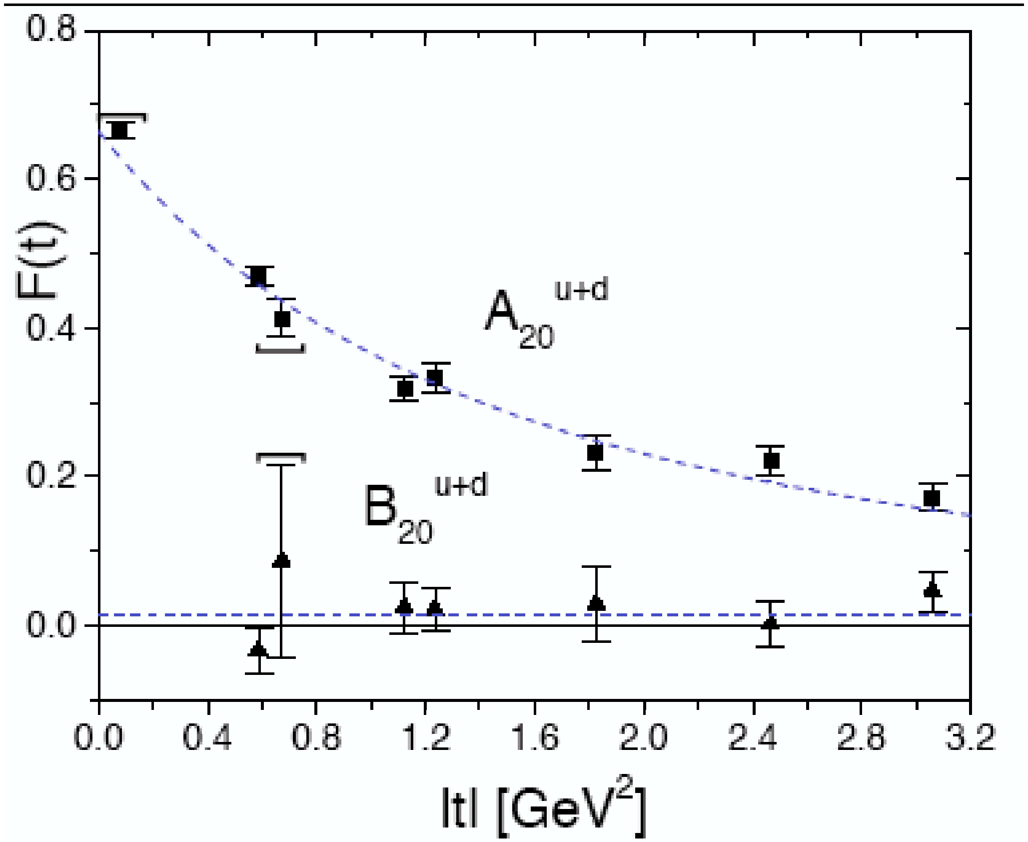
Generalized Form Factors

- $\overline{\psi}\gamma^{\{\mu}D^{\nu\}}\psi$ has 3 GFFs: A_{20}, B_{20}, C_2



Quark Angular Momentum: $m_\pi = 897$MeV

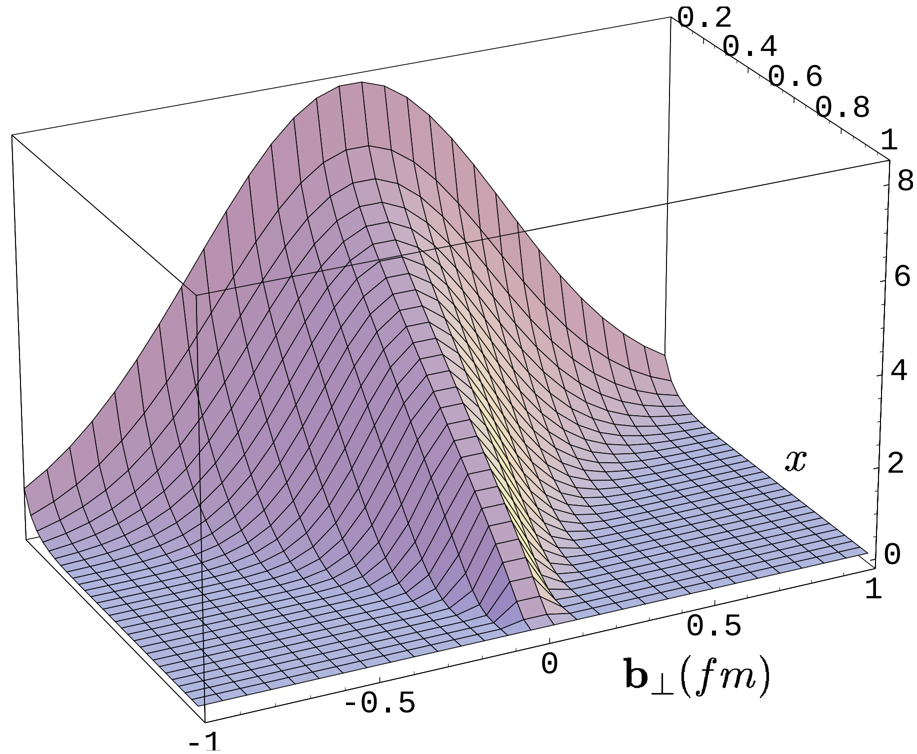
- $\frac{1}{2}\Delta\Sigma_{u+d} = \frac{1}{2}\tilde{A}_{10}^{u+d}(0) = \frac{1}{2}\left(\langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d}\right) \sim \frac{1}{2}0.682(18)$
- $J_{u+d} = \frac{1}{2}\left(A_{20}^{u+d}(0) + B_{20}^{u+d}(0)\right) \sim \frac{1}{2}0.675(7)$
- $L_{u+d} = J_{u+d} - \frac{1}{2}\Delta\Sigma_{u+d} \sim 0 \quad \Rightarrow \quad J_g = \frac{1}{2}0.32$



QCDSF hep-ph/0304249; Mathur hep-ph/9912289; Gadiyak hep-lat/0112040

Transverse Structure

- 3D quark distribution: $q(x, \vec{b}_\perp)$
- x is longitudinal momentum fraction
- $\vec{b}_\perp$ is transverse displacement of active parton relative to nucleon center of longitudinal momentum
- simple model calculation



Burkardt hep-ph/0207047

Transverse Structure

$$H_q(x,0,-\vec{\Delta}_\perp^2) = \int d^2b_\perp \; e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} q(x,\vec{b}_\perp)$$

$$A^q_{n0}(t) = \int_{-1}^{1} dx \; x^{n-1} H_q(x,0,t)$$

- at $x = 1$ a single quark carries all the momentum

$$\lim_{x\to 1} q(x,\vec{b}_\perp) \propto \delta^2(\vec{b}_\perp) \qquad H_q(1,0,t) = \text{constant}$$

- higher moments A^q_{n0} weight $x \sim 1$ more heavily

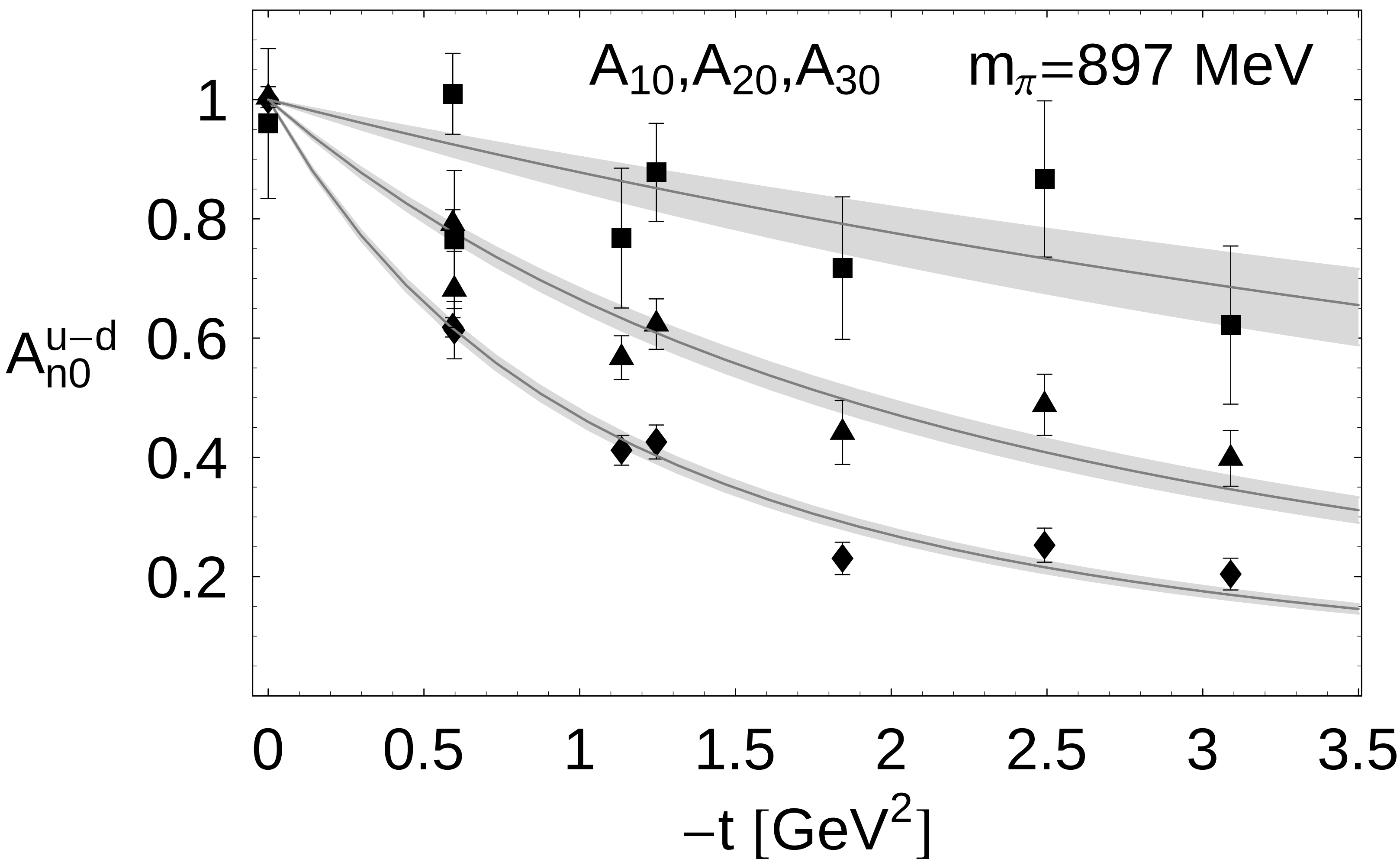
$$\lim_{n\to\infty} A^q_{n0}(t) \propto H_q(1,0,t)$$

- slopes of A^q_{n0} should decrease as n increases

- A_{10}, A_{30}, $\tilde{A}_{20}$ measure $q - \overline{q}$ & $\tilde{A}_{10}$, $\tilde{A}_{30}$, A_{20} measure $q + \overline{q}$

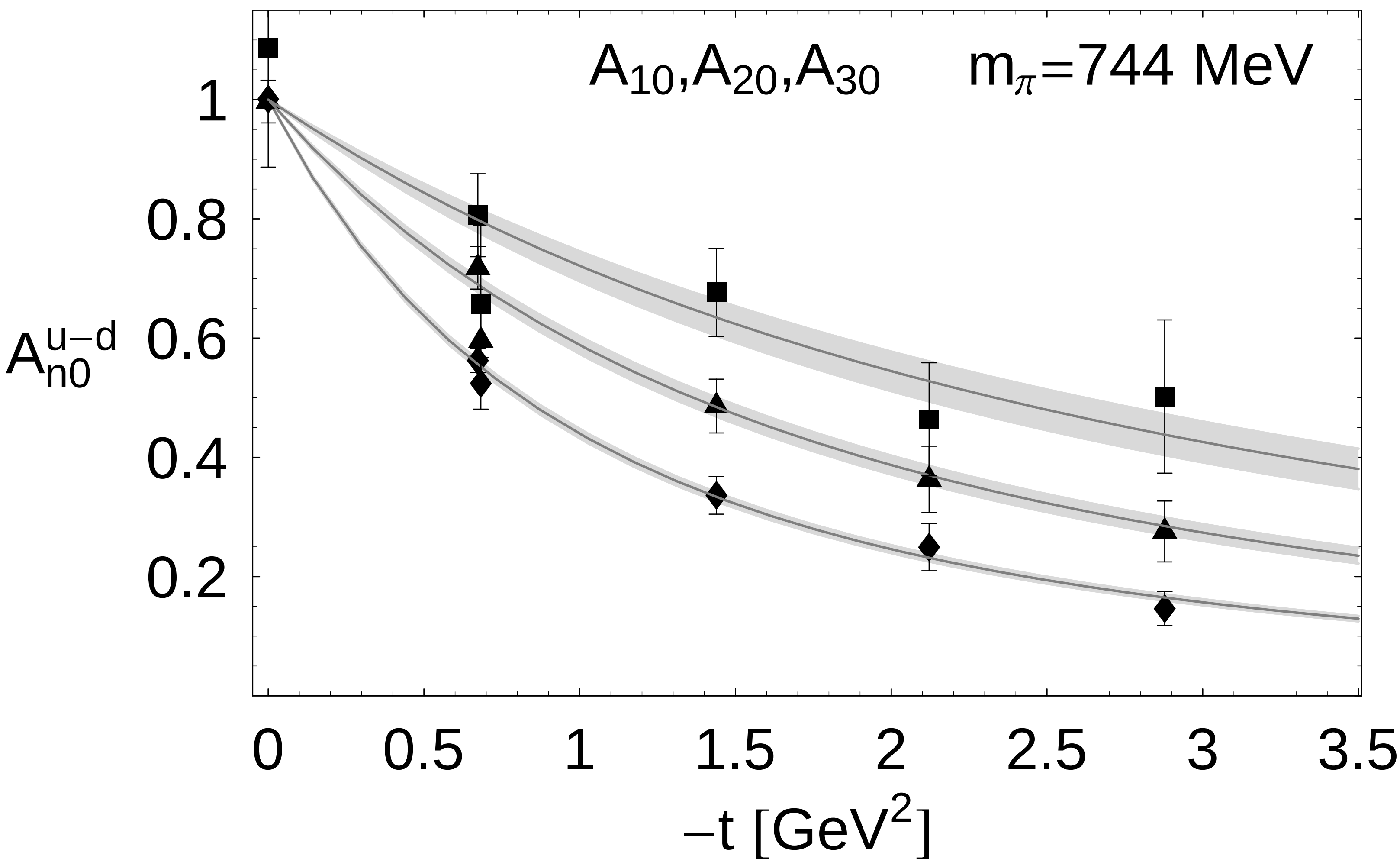
Transverse Structure: $m_\pi = 897$MeV

- slope of $A_{10}^{u-d} = -0.93 \pm 0.04$ (GeV)$^{-2}$
- slope of $A_{30}^{u-d} = -0.13 \pm 0.03$ (GeV)$^{-2}$ (factor of 7)



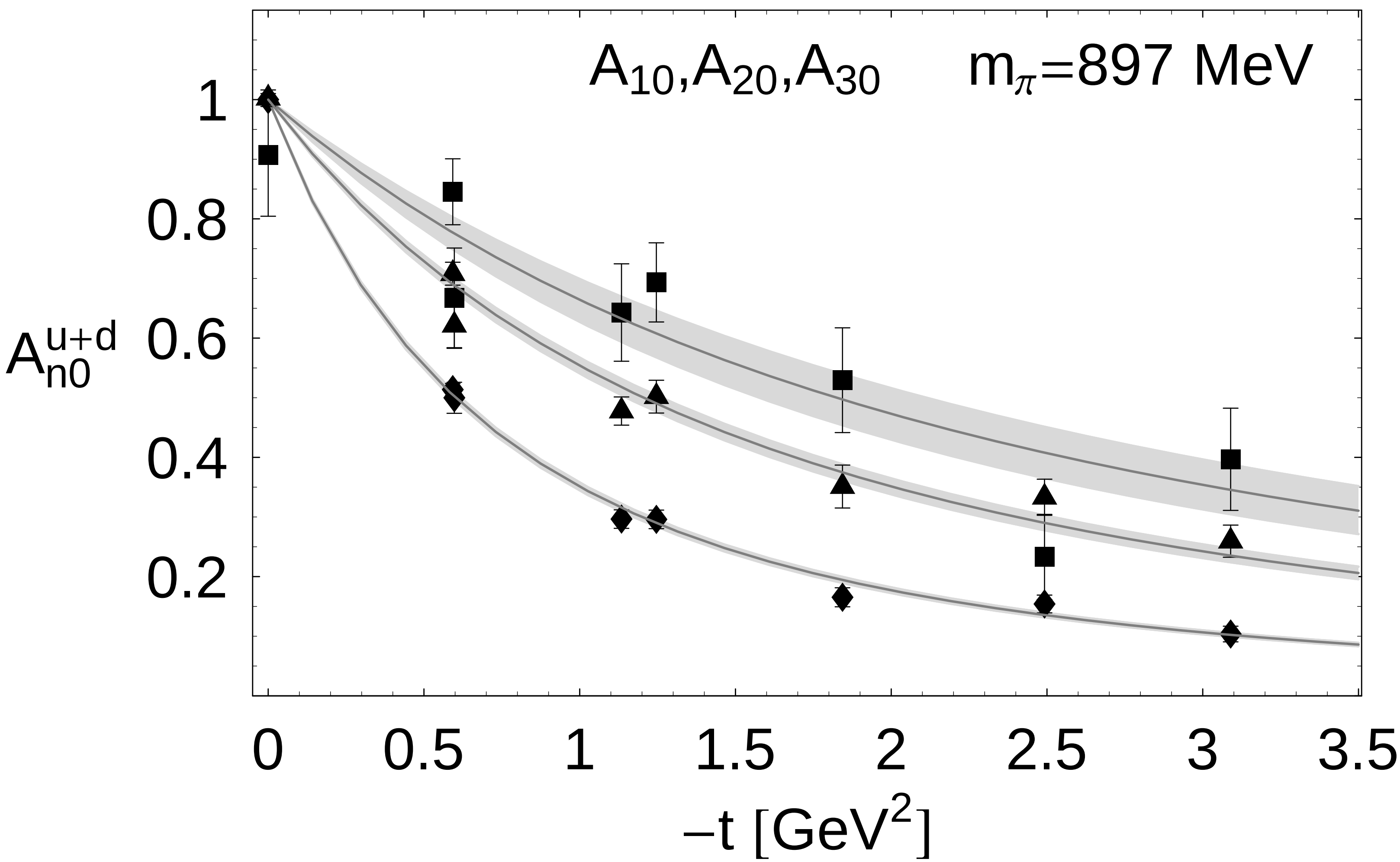
Transverse Structure: $m_\pi = 744\text{MeV}$

- slope of $A_{10}^{u-d} = -1.02 \pm 0.03$ (GeV)$^{-2}$
- slope of $A_{30}^{u-d} = -0.36 \pm 0.04$ (GeV)$^{-2}$ (factor of 3)



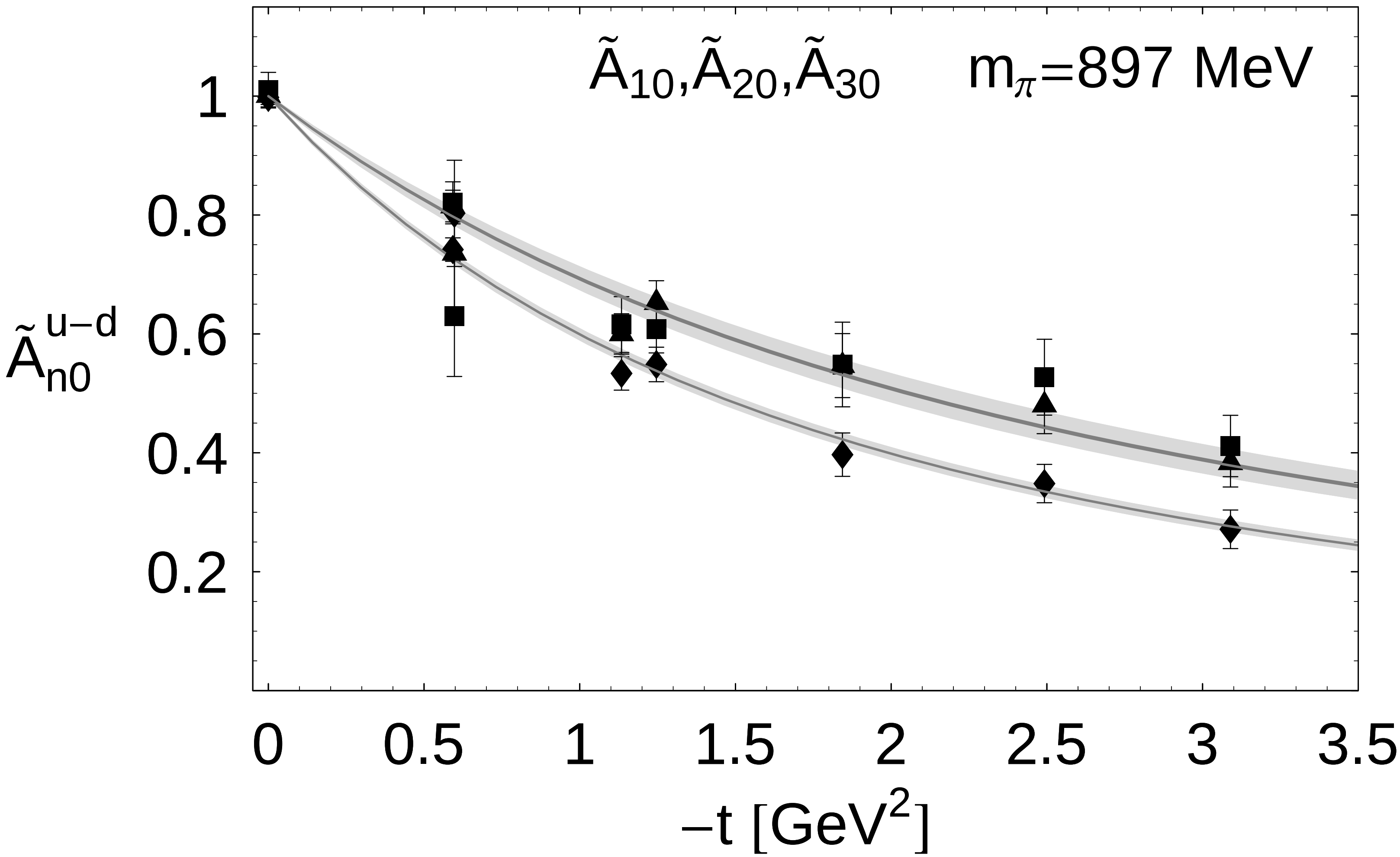
Transverse Structure: Flavor Dependence

- slope of $A_{10}^{u+d} = -1.38 \pm 0.03\ (\mathrm{GeV})^{-2}$
- slope of $A_{30}^{u+d} = -0.45 \pm 0.07\ (\mathrm{GeV})^{-2}$ (factor of 3)



Transverse Structure: Spin Dependence

- slope of $\tilde{A}_{10}^{u-d} = -0.58 \pm 0.02$ (GeV)$^{-2}$
- slope of $\tilde{A}_{30}^{u-d} = -0.40 \pm 0.03$ (GeV)$^{-2}$ (factor of 1.5)



Transverse Structure

- transverse rms radius

$$\left\langle b_\perp^2 \right\rangle_x = \frac{\int d^2 b_\perp \, b_\perp^2 \, q(x, \vec{b}_\perp)}{\int d^2 b_\perp \, q(x, \vec{b}_\perp)}$$

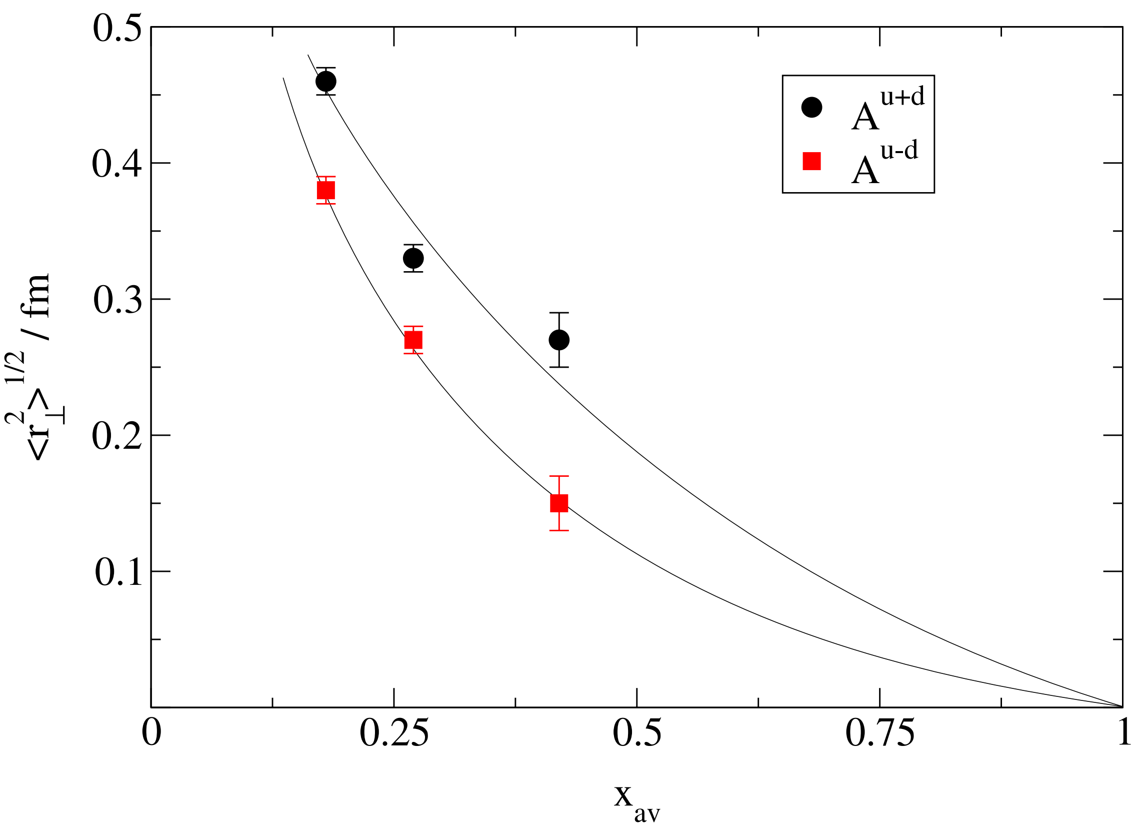
- smeared transverse rms radius

$$\frac{A'_{n0}(0)}{A_{n0}(0)} = -\frac{1}{4} \left\langle b_\perp^2 \right\rangle_{(n)} \qquad \left\langle b_\perp^2 \right\rangle_{(n)} = \frac{\int d^2 b_\perp \, b_\perp^2 \, \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)}{\int d^2 b_\perp \, \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)}$$

- the average x in $\left\langle b_\perp^2 \right\rangle_{(n)}$

$$x_{\text{av}}^{(n)} = \frac{\int d^2 b_\perp \, \int_{-1}^{1} dx \, x \cdot x^{n-1} q(x, \vec{b}_\perp)}{\int d^2 b_\perp \, \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)} = \frac{< x^n >}{< x^{n-1} >}$$

Smeared Transverse Structure



- charge radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{\text{charge}}} = 0.42$ fm (0.72 fm from experiment), no pion cloud
- non-singlet radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{u-d}^{(1)}} = 0.38$ fm & $\sqrt{\left\langle b_\perp^2 \right\rangle_{u-d}^{(3)}} = 0.15$ fm, 61% decrease
- singlet radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{u+d}^{(1)}} = 0.46$ fm & $\sqrt{\left\langle b_\perp^2 \right\rangle_{u+d}^{(3)}} = 0.27$ fm, 41% decrease

Current Work: Chiral Limit

- hybrid calculation: Asqtad staggered sea quarks (MILC) & domain wall valence quarks with HYP smearing
- $a = 0.13$ fm, $L = 2.6$ fm, $20^3 \times 32$
- $M_\pi = 343, 635$ MeV
- O(100) gluon configurations
- extended statistics, intermediate masses, volume dependence

Summary

- generalized parton distributions:
  (1) contain form factors and ordinary parton distributions
  (2) determine the spin decomposition of the nucleon
  (3) measure Fourier transform of transverse quark distribution

- building blocks method to determine all matrix elements for parton distributions, form factors, and generalized parton distributions

- overdetermined observables method to accurately measure generalized form factors upto 3 GeV^2

- in heavy pion world: $\frac{1}{2} = \frac{1}{2}(0.68 + 0.0 + 0.32)$ & observed scaling in $\frac{F_2}{F_1}$

- observed significant dependence of slopes of generalized form factors A_{n0} on n indicating a significant variation of transverse size with x