

# Generalized Parton Distributions from Lattice QCD

Dru B. Renner  
University of Arizona

APS Topical Group on Hadronic Physics

October 25, 2004

Generalized parton distributions determine the transverse distribution of partons within the nucleon as well as the fraction of the nucleon spin carried by the orbital angular momentum and helicity of the partons. Lattice QCD calculations of the generalized parton distributions will provide an essential complement to the experimental determination of the nucleon's structure. In particular lattice QCD calculations provide insight into the transverse distribution of quarks within the nucleon and additionally determine the elastic form factors, the spin decomposition of the nucleon, and low moments of the ordinary parton distributions. I will review our ongoing lattice QCD calculations of the generalized parton distributions of the nucleon and highlight those aspects of nucleon structure best illuminated by lattice QCD.

# Generalized Parton Distributions from Lattice QCD

Dru B. Renner  
University of Arizona

APS Topical Group on Hadronic Physics  
October 25, 2004

Lattice Hadron Physics Collaboration

R. Edwards  
G. Fleming  
Ph. Hägler  
J. Negele

K. Orginos  
A. Pochinsky  
D. Richards  
W. Schroers

[http://talks.drubryantrenner.org/ghp2004\\_10-25-04.pdf](http://talks.drubryantrenner.org/ghp2004_10-25-04.pdf)

# Generalized Parton Distributions from Lattice QCD

What new physics do we learn?

- Spin Decomposition - decomposition of nucleon spin into quark helicity, quark orbital, and gluon contributions

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma_{u+d} + L_{u+d} + J_g$$

- Transverse Structure - 3D distribution of quarks in a mixed representation: 2 transverse coordinates  $\vec{b}_\perp$  and 1 longitudinal momentum  $x$

$$q(x, \vec{b}_\perp) \quad \text{and} \quad \Delta q(x, \vec{b}_\perp)$$

What matrix elements determine the quark angular momenta  
and transverse quark distributions?

# Generalized Form Factors

- unpolarized and polarized twist two operators

$$O_q^{\mu_1 \dots \mu_n} = \bar{q} i D^{(\mu_1} \dots i D^{\mu_{n-1}} \gamma^{\mu_n)} q$$

$$\tilde{O}_q^{\mu_1 \dots \mu_n} = \bar{q} i D^{(\mu_1} \dots i D^{\mu_{n-1}} \gamma^{\mu_n)} \gamma^5 q$$

- off-forward matrix elements of the twist two operators [1]

$$\begin{aligned} \langle P', S' | O_q^{\mu_1 \dots \mu_n} | P, S \rangle &= \bar{U}(P', S') \left[ \sum_{\substack{i=0 \\ \text{even}}}^{n-1} A_{ni}^q(t) K_{ni}^A(P', P) \right. \\ &\quad \left. + \sum_{\substack{i=0 \\ \text{even}}}^{n-1} B_{ni}^q(t) K_{ni}^B(P', P) + \delta_{\text{even}}^n C_n^q(t) K_n^C(P', P) \right] U(P, S) \end{aligned}$$

$$\begin{aligned} \langle P', S' | \tilde{O}_q^{\mu_1 \dots \mu_n} | P, S \rangle &= \bar{U}(P', S') \left[ \sum_{\substack{i=0 \\ \text{even}}}^{n-1} \tilde{A}_{ni}^q(t) \tilde{K}_{ni}^A(P', P) + \right. \\ &\quad \left. \sum_{\substack{i=0 \\ \text{even}}}^{n-1} \tilde{B}_{ni}^q(t) \tilde{K}_{ni}^B(P', P) \right] U(P, S) \end{aligned}$$

## Basic Properties of Generalized Form Factors

- moments of parton distributions -  $\langle P|O_q^{\mu_1 \dots \mu_n}|P\rangle$  and  $\langle P|\tilde{O}_q^{\mu_1 \dots \mu_n}|P\rangle$

$$A_{n0}^q(0) = \int_{-1}^1 dx x^{n-1} q(x) \quad \text{and} \quad \tilde{A}_{n0}^q(0) = \int_{-1}^1 dx x^{n-1} \Delta q(x)$$

- form factors -  $O_q^\mu = \bar{q}\gamma^\mu q$  and  $\tilde{O}_q^\mu = \bar{q}\gamma^\mu\gamma^5 q$

$$A_{10}^q(t) = F_1^q(t) \quad \text{and} \quad B_{10}^q(t) = F_2^q(t)$$

$$\tilde{A}_{10}^q(t) = G_A^q(t) \quad \text{and} \quad \tilde{B}_{10}^q(t) = G_P^q(t)$$

# Quark Angular Momenta and Transverse Quark Distributions

- quark angular momenta [1]

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma^{u+d} + L^{u+d} + J^g$$

$$\Delta \Sigma^q = \tilde{A}_{10}^q(0) \quad J^q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0)) \quad L^q = J^q - \frac{1}{2} \Delta \Sigma^q$$

- transverse quark distributions [2]

$$\int_{-1}^1 dx x^{n-1} q(x, \vec{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} A_{n0}^q(-\vec{\Delta}_\perp^2)$$

$$\int_{-1}^1 dx x^{n-1} \Delta q(x, \vec{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \tilde{A}_{n0}^q(-\vec{\Delta}_\perp^2)$$

[1] X. D. Ji hep-ph/9603249

[2] M. Burkardt hep-ph/0005108

## Calculations with Heavy-*ish* Quarks

- Wilson quarks
- flavors:  $N_F = 2$
- lattice spacing:  $a = 0.095$  fm
- lattice size:  $L = 1.52$  fm
- pion masses:  $M_\pi = 753(10), 835(13), 895(15)$  MeV

What do we learn about the nature of the nucleon spin?

# Quark Angular Momenta

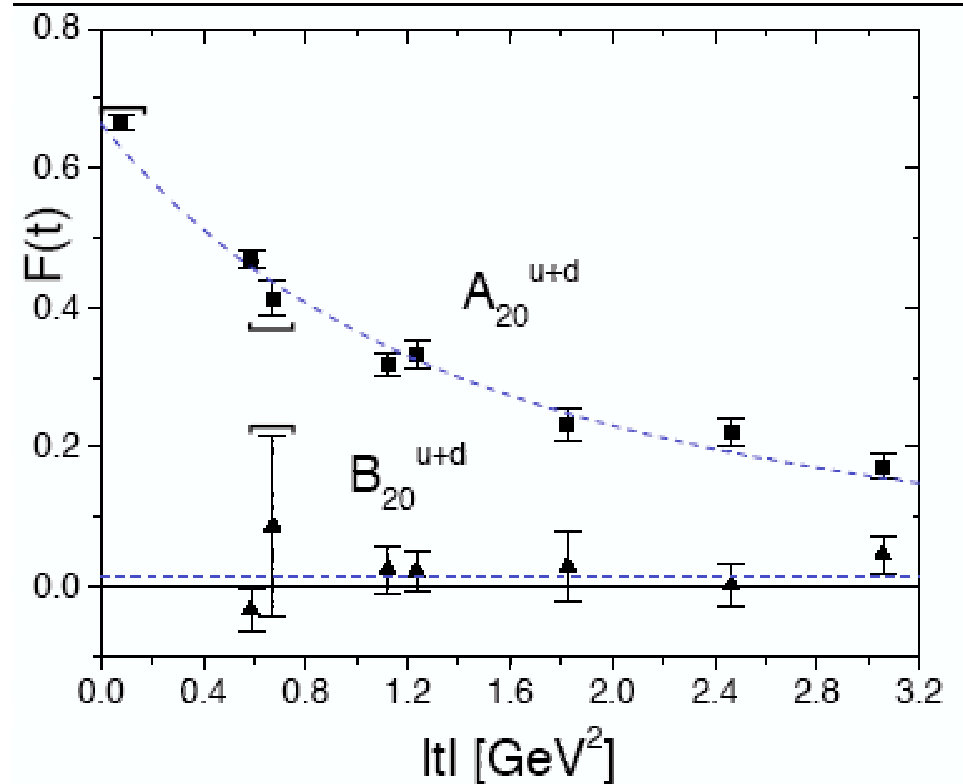
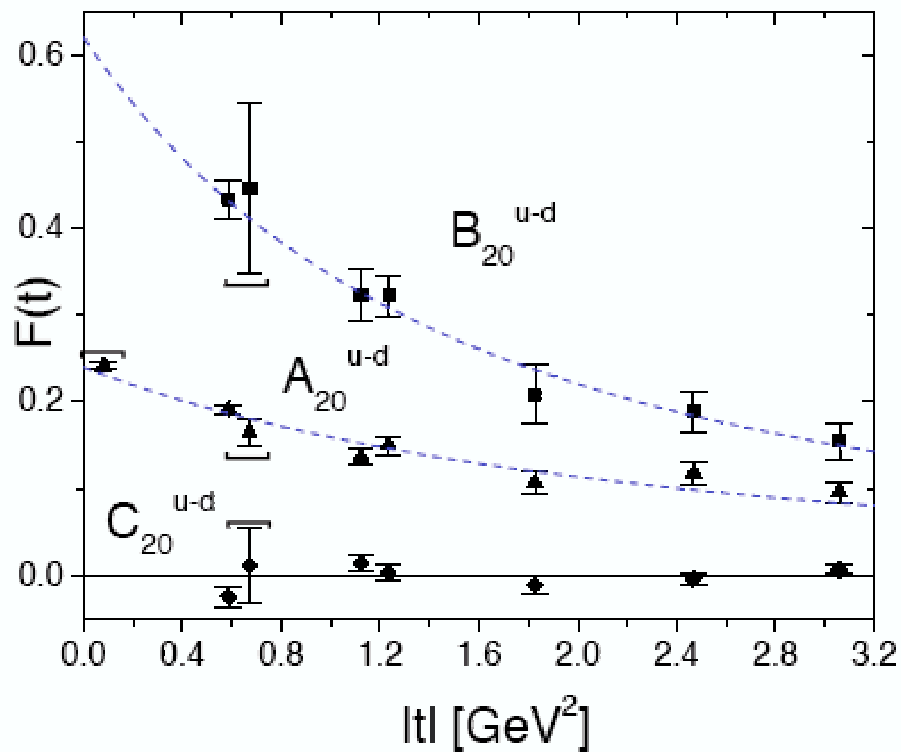
$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma^{u+d} + L^{u+d} + J_g$$

$$\Delta \Sigma_q = \frac{1}{2} \tilde{A}_{10}^q(0)$$

$$J_q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0))$$

$$L_q = J_q - \frac{1}{2} \Delta \Sigma_q$$

$$J_g = \frac{1}{2} - J_{u+d}$$



## Quark Angular Momenta: $m_\pi = 895$ MeV

- quark helicity, quark orbital, and gluon contributions (modulo disconnected diagrams)

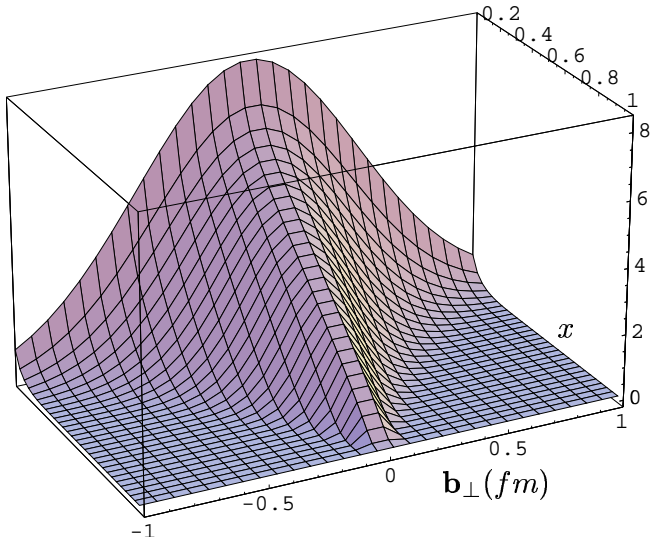
$$2 \cdot \frac{1}{2} = \begin{array}{ccc} \Delta\Sigma^{u+d} & + & 2L^{u+d} & + & 2J^g \\ +0.682(20) & & -0.002(3) & & + 0.320(16) \end{array}$$

- quark orbital motion,  $L^{u-d} = -0.193(32)$

$$2L_{\min}^q = 2 \cdot \left( \frac{|L^u| + |L^d|}{2} \right) \geq |L^{u-d}| = 0.193(32)$$

What do we learn about the transverse quark structure of the nucleon?

# Transverse Distributions



$$A_{n0}^q(-\vec{\Delta}_\perp^2) = \int d^2b_\perp e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_\perp)$$

$$\langle b_\perp^2 \rangle_{(n)}^q = -4 \frac{A_{n0}^{q'}(0)}{A_{n0}^q(0)}$$

- at  $x = 1$  a single quark carries all the momentum

$$\lim_{x \rightarrow 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

- higher moments  $A_{n0}^q$  weight  $x \sim 1$  more heavily

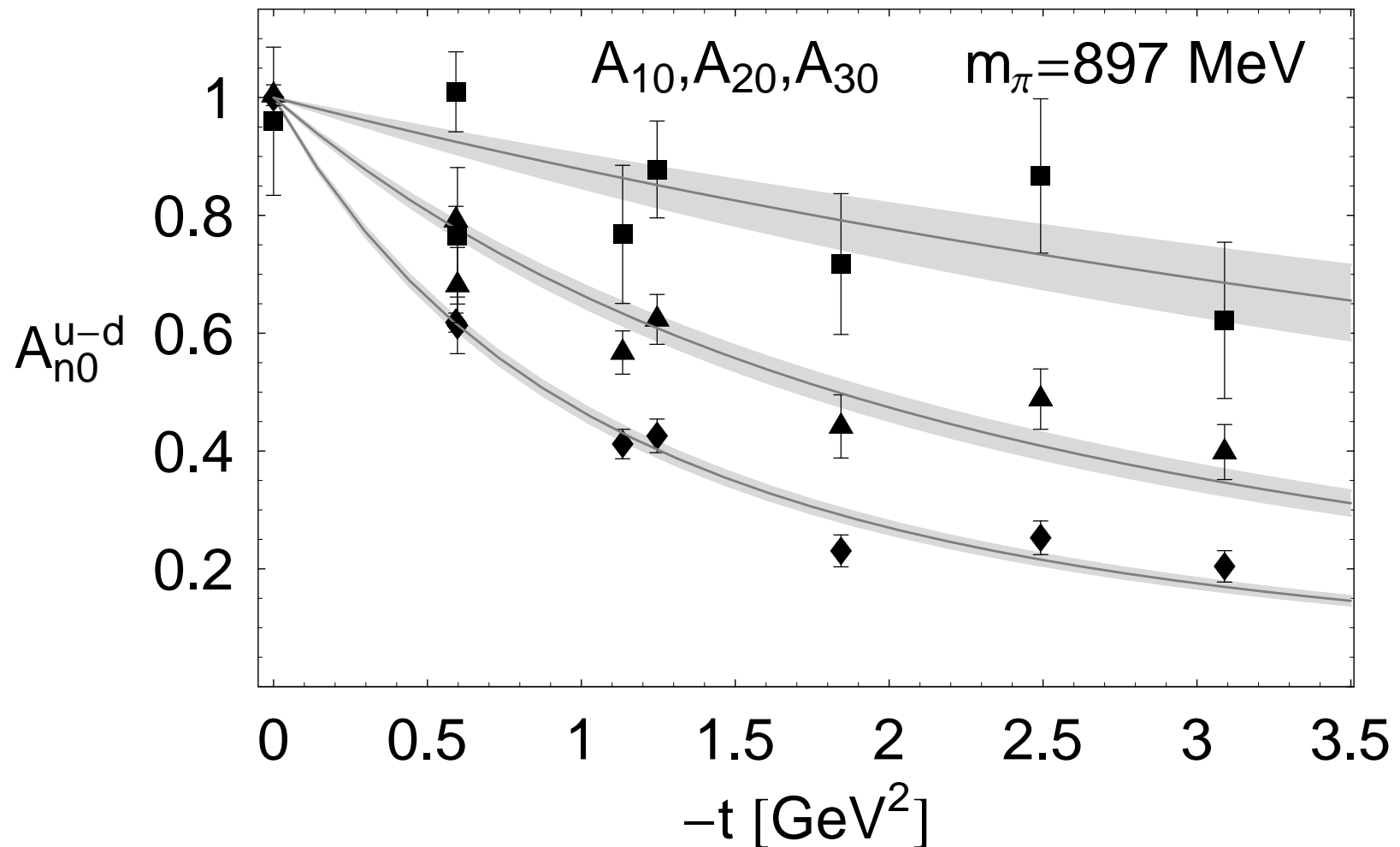
$$\lim_{n \rightarrow \infty} A_{n0}^q(t) \propto \int d^2b_\perp e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \delta^2(\vec{b}_\perp) = \text{constant}$$

- slopes of  $A_{n0}^q$  should decrease as  $n$  increases

- $A_{10}, A_{30}, \tilde{A}_{20}$  measure  $q - \bar{q}$  &  $\tilde{A}_{10}, \tilde{A}_{30}, A_{20}$  measure  $q + \bar{q}$

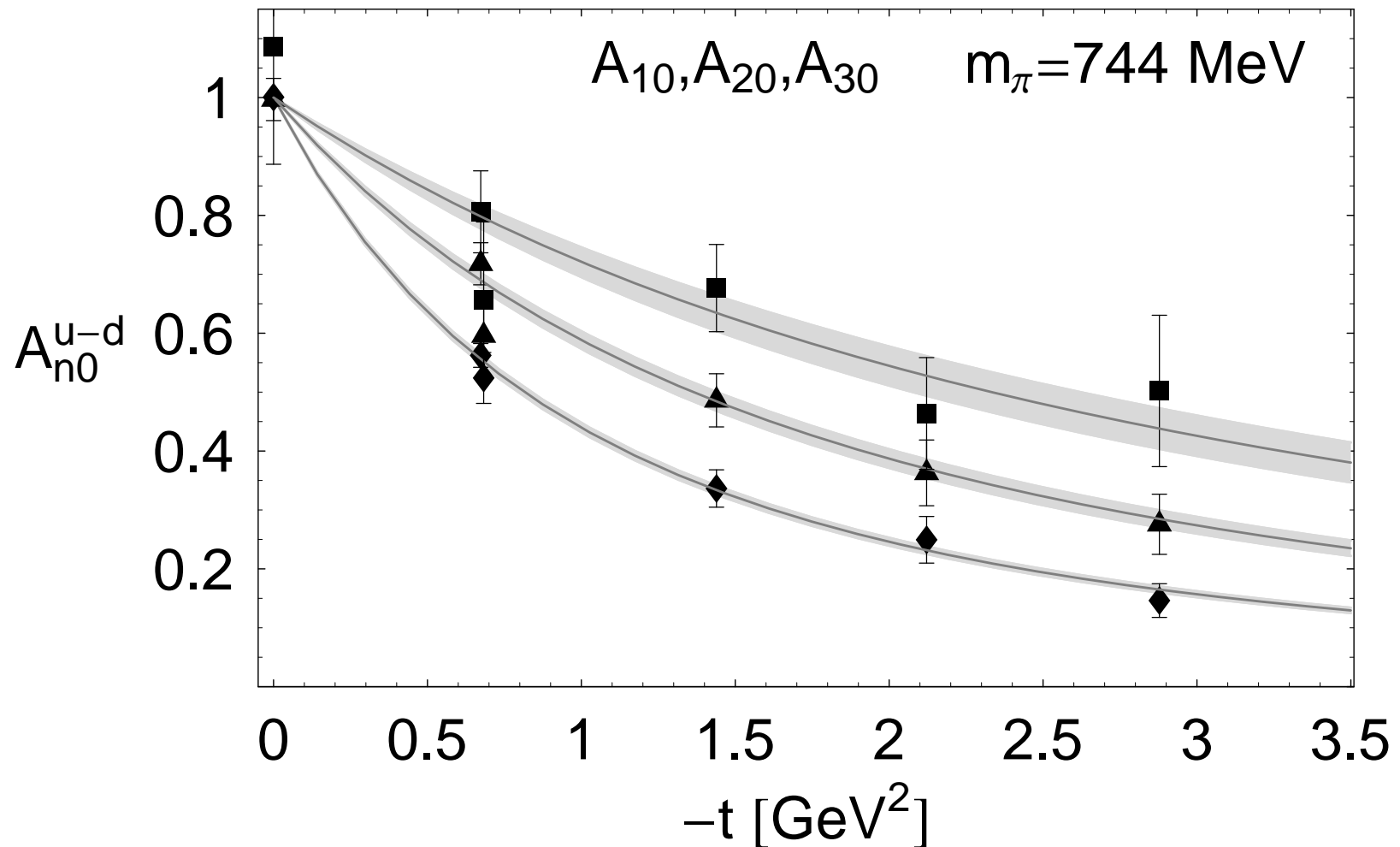
## Transverse Distributions: $m_\pi = 897$ MeV

- slope of  $A_{10}^{u-d} = -0.93 \pm 0.04$  (GeV)<sup>-2</sup>
- slope of  $A_{30}^{u-d} = -0.13 \pm 0.03$  (GeV)<sup>-2</sup> (factor of 7)



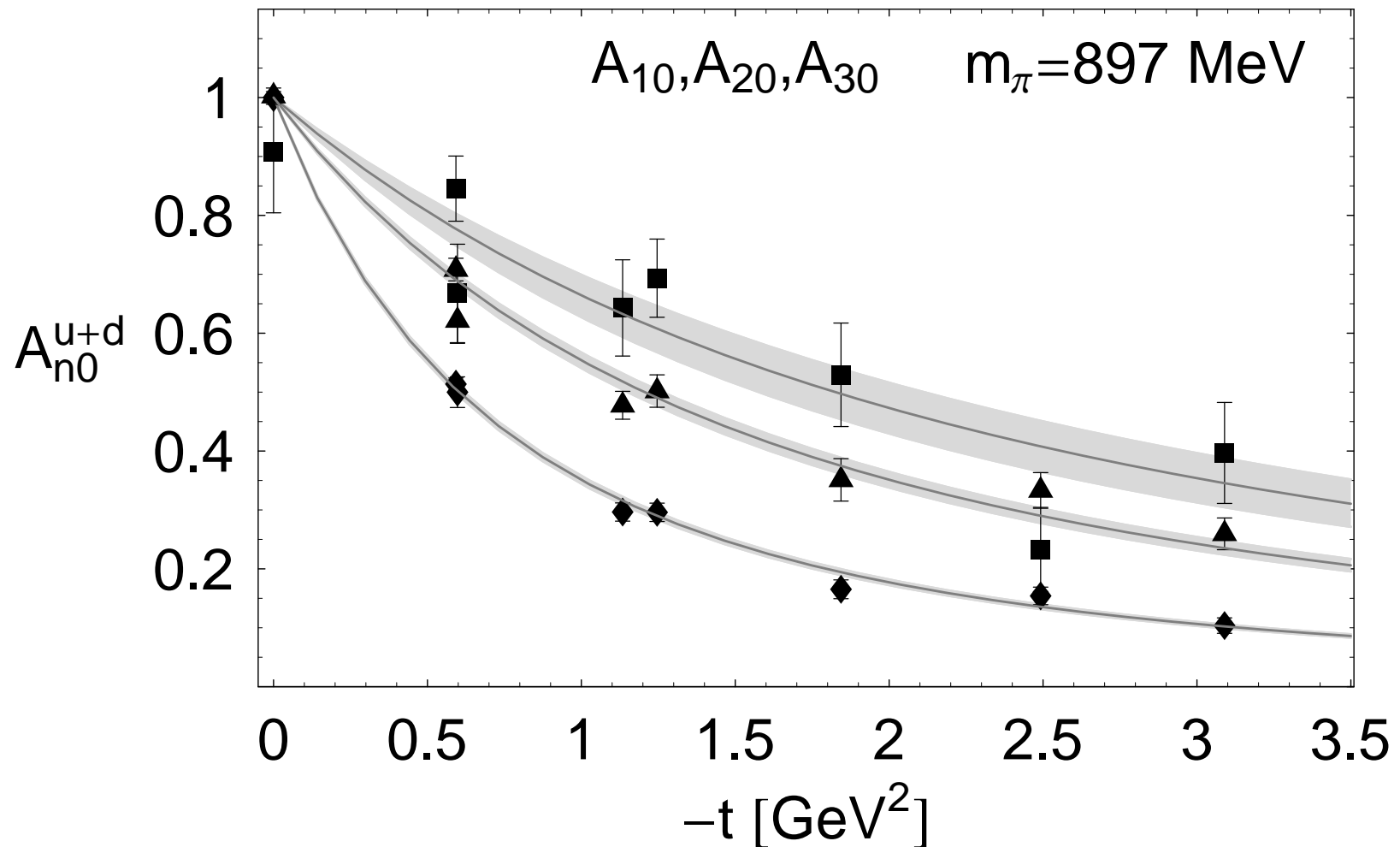
## Transverse Distributions: Mass Dependence

- slope of  $A_{10}^{u-d} = -1.02 \pm 0.03 \text{ (GeV)}^{-2}$
- slope of  $A_{30}^{u-d} = -0.36 \pm 0.04 \text{ (GeV)}^{-2}$  (factor of 3)



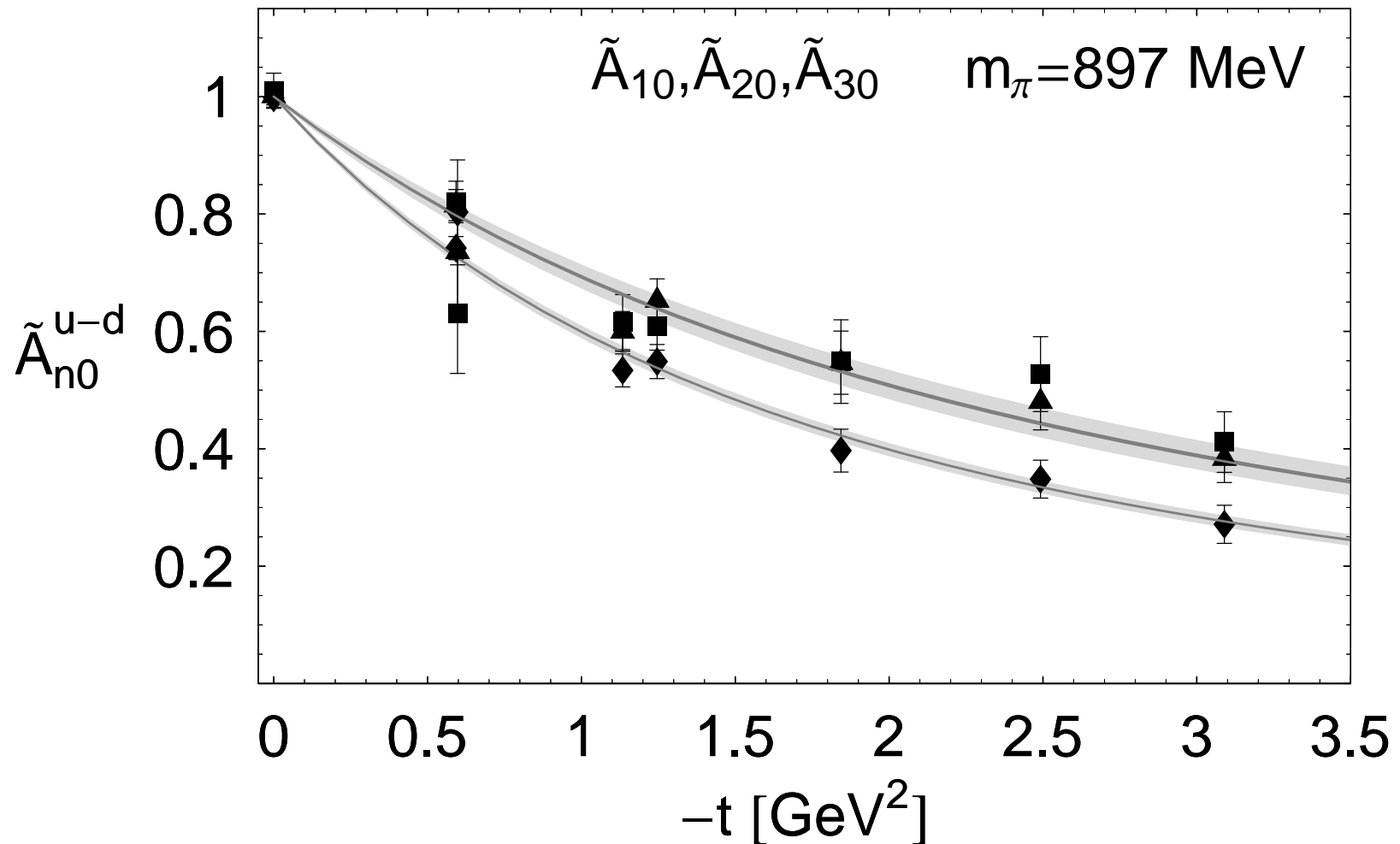
## Transverse Distributions: Flavor Dependence

- slope of  $A_{10}^{u+d} = -1.38 \pm 0.03 \text{ (GeV)}^{-2}$
- slope of  $A_{30}^{u+d} = -0.45 \pm 0.07 \text{ (GeV)}^{-2}$  (factor of 3)



## Transverse Distributions: Spin Dependence

- slope of  $\tilde{A}_{10}^{u-d} = -0.58 \pm 0.02 \text{ (GeV)}^{-2}$
- slope of  $\tilde{A}_{30}^{u-d} = -0.40 \pm 0.03 \text{ (GeV)}^{-2}$  (factor of 1.5)



## Transverse Distributions: $x$ Dependence

- transverse rms radius

$$\langle b_{\perp}^2 \rangle_x = \frac{\int d^2 b_{\perp} b_{\perp}^2 q(x, \vec{b}_{\perp})}{\int d^2 b_{\perp} q(x, \vec{b}_{\perp})}$$

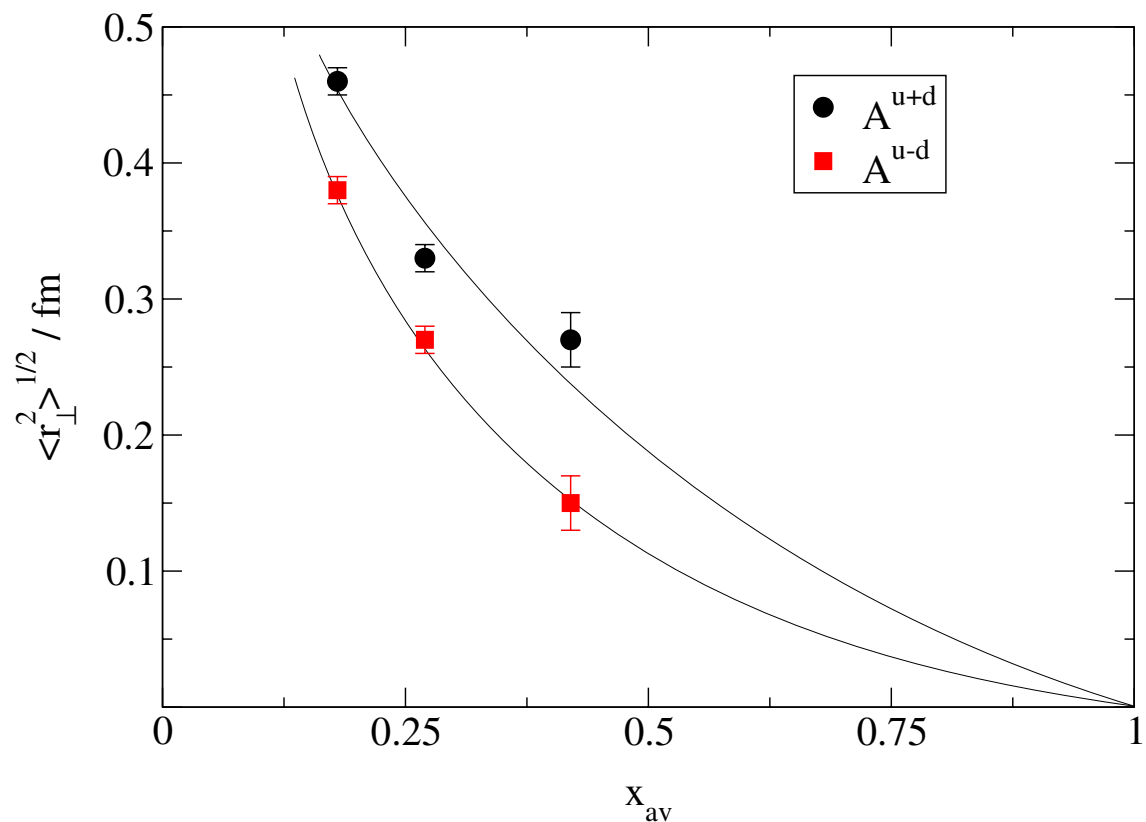
- transverse rms *moment* radius

$$\frac{A'_{n0}(0)}{A_{n0}(0)} = -\frac{1}{4} \langle b_{\perp}^2 \rangle_{(n)} \quad \langle b_{\perp}^2 \rangle_{(n)} = \frac{\int d^2 b_{\perp} b_{\perp}^2 \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_{\perp})}{\int d^2 b_{\perp} \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_{\perp})}$$

- the average  $x$  in  $\langle b_{\perp}^2 \rangle_{(n)}$

$$x_{\text{av}}^{(n)} = \frac{\int d^2 b_{\perp} \int_{-1}^1 dx x \cdot x^{n-1} q(x, \vec{b}_{\perp})}{\int d^2 b_{\perp} \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_{\perp})} = \frac{\langle x^n \rangle}{\langle x^{n-1} \rangle}$$

## Transverse Distributions: $x$ Dependence

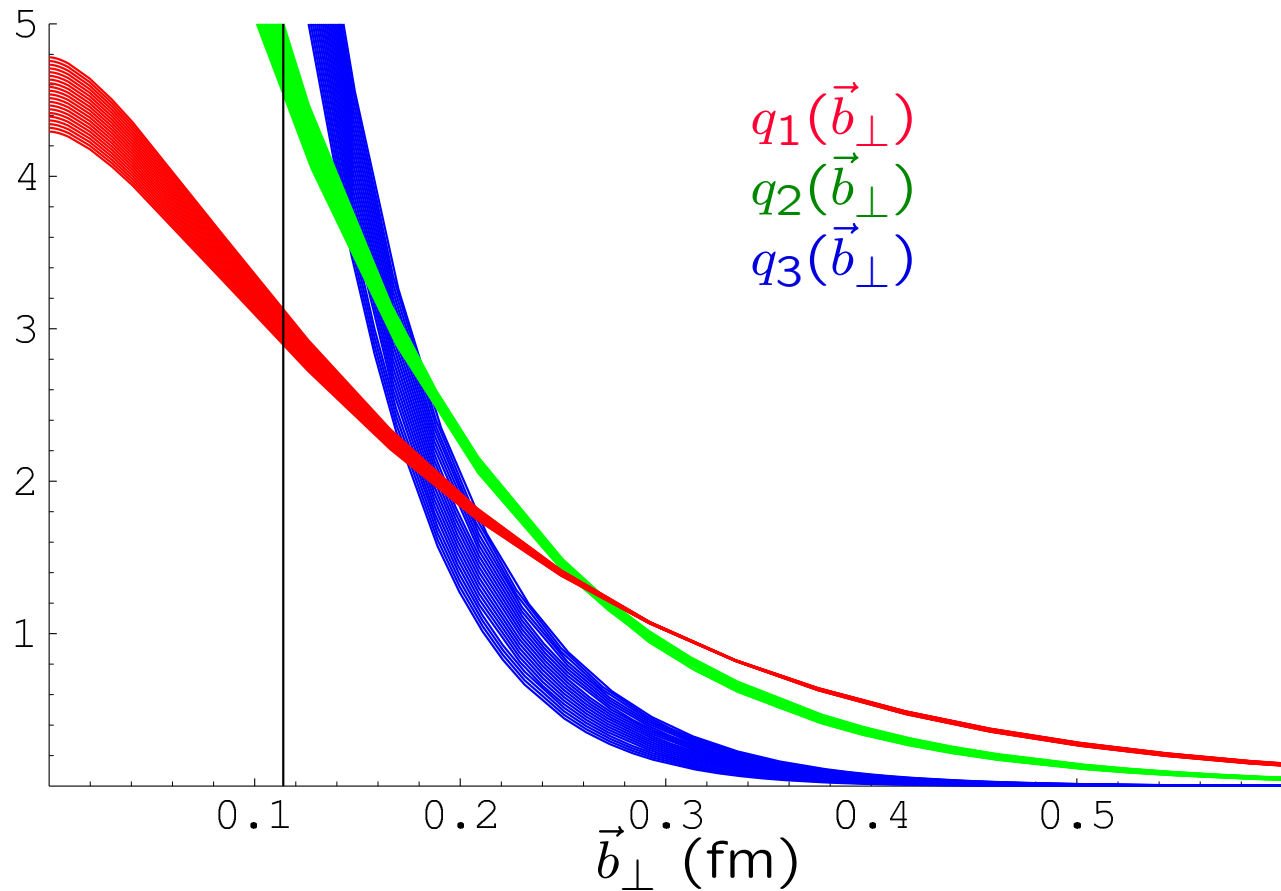


- non-singlet radius:  $\sqrt{\langle b_{\perp}^2 \rangle_{u-d}^{(1)}} = 0.38 \text{ fm}$  &  $\sqrt{\langle b_{\perp}^2 \rangle_{u-d}^{(3)}} = 0.15 \text{ fm}$ ,  
61% decrease
- singlet radius:  $\sqrt{\langle b_{\perp}^2 \rangle_{u+d}^{(1)}} = 0.46 \text{ fm}$  &  $\sqrt{\langle b_{\perp}^2 \rangle_{u+d}^{(3)}} = 0.27 \text{ fm}$ ,  
41% decrease

## Transverse Distributions: $\vec{b}_\perp$ Dependence

$$q_1(\vec{b}_\perp) = \int_{-1}^1 dx q(x, \vec{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} A_{10}^q(-\vec{\Delta}_\perp^2)$$

$$q_2(\vec{b}_\perp) = \int_{-1}^1 dx x q(x, \vec{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} A_{20}^q(-\vec{\Delta}_\perp^2)$$



## Conclusions

- unambiguous observation of quark orbital motion,  $L^{u-d} = -0.193(32)$ , for even heavy pion masses
- the transverse size of the nucleon,  $\sqrt{\langle r_{\perp}^2 \rangle}$ , in the heavy pion world, shows a significant dependence on the longitudinal momentum fraction  $\langle x \rangle$

## Moments of Generalized Parton Distributions

- moments of generalized parton distributions

$$\int_{-1}^1 dx x^{n-1} H_q(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^{n-1} A_{ni}^q(t) (-2\xi)^i + \delta_{\text{even}}^n C_n^q(t) (-2\xi)^n$$

$$\int_{-1}^1 dx x^{n-1} E_q(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^{n-1} B_{ni}^q(t) (-2\xi)^i - \delta_{\text{even}}^n C_n^q(t) (-2\xi)^n$$

- transverse momentum transfer,  $\xi \rightarrow 0$

$$\int_{-1}^1 dx x^{n-1} H_q(x, 0, t) = A_{n0}^q(t)$$

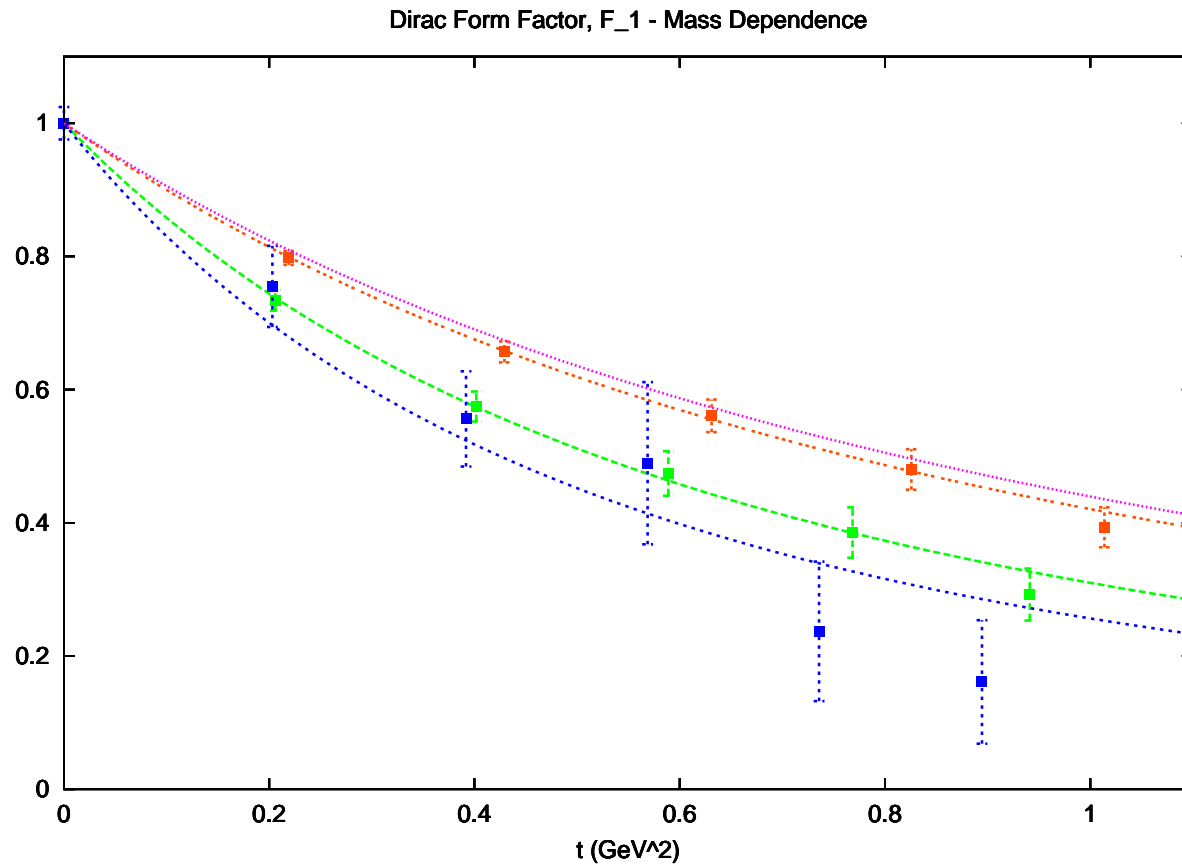
$$\int_{-1}^1 dx x^{n-1} E_q(x, 0, t) = B_{n0}^q(t)$$

- similar results relating the polarized GPDs,  $\tilde{H}_q(x, 0, t)$  and  $\tilde{E}_q(x, 0, t)$ , to the polarized GFFs,  $\tilde{A}_{n0}^q(t)$  and  $\tilde{B}_{n0}^q(t)$

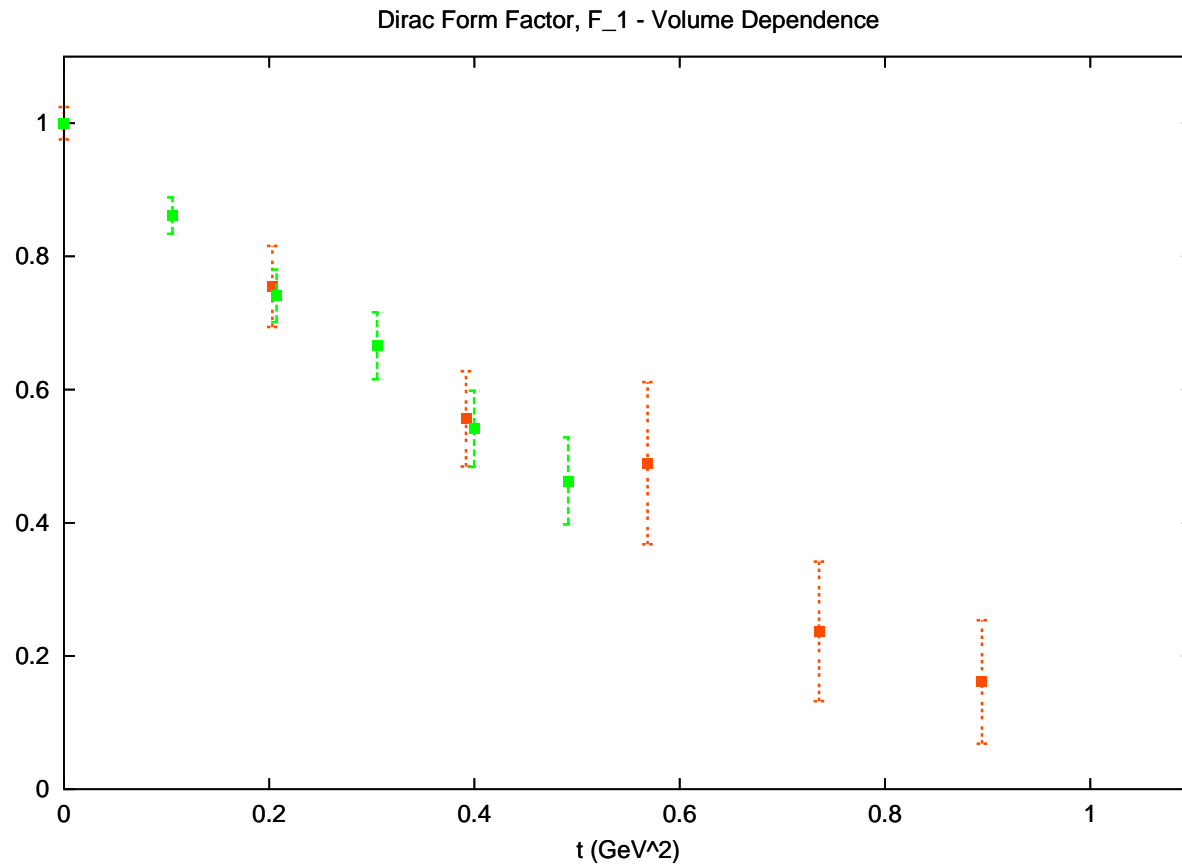
## Calculations with Light-*ish* Quarks

- Kogut-Susskind sea quarks and domain wall valence quarks
- flavors:  $N_F = 2 + 1$
- lattice spacing:  $a = 0.13$  fm
- lattice sizes:  $L = 2.6$  fm and  $L = 3.6$  fm
- pion masses:  $M_\pi = 329(15), 564(2), 730(3)$  MeV

# $F_1$ Form Factor - Mass Dependence

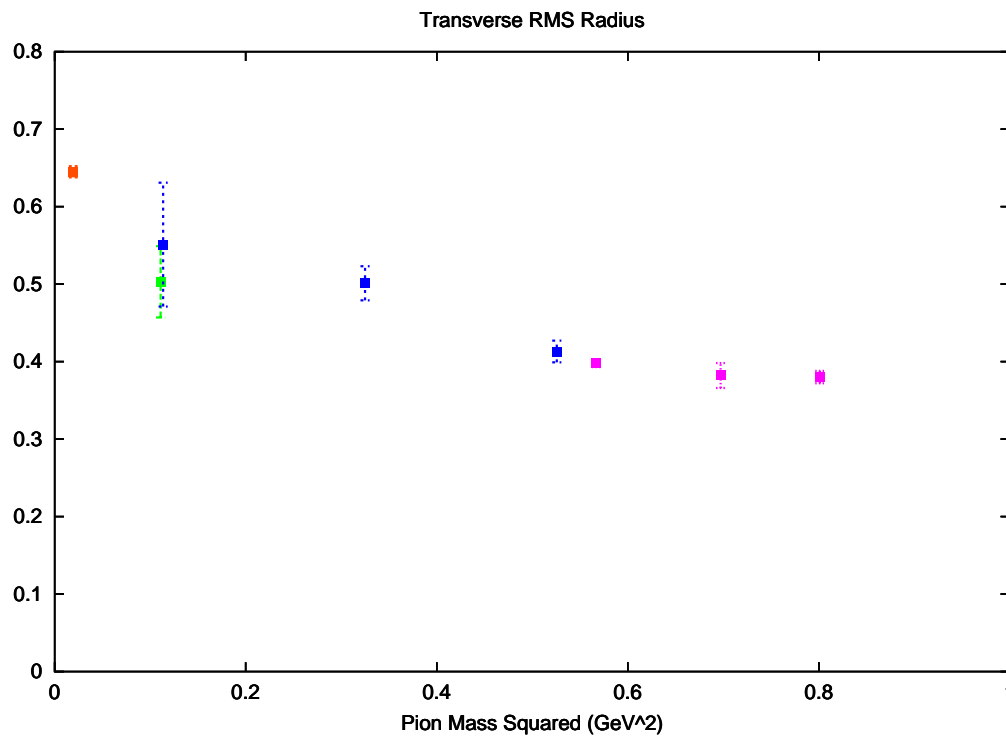


# $F_1$ Form Factor - Volume Dependence



# Transverse RMS Radius $\sqrt{r_{\perp}^2}$

$m_{\pi}^{\text{DWF}}$	$L$ (fm)	$\sqrt{r_{\perp}^2}$ (fm)
725	2.6	0.413(14)
570	2.6	0.501(22)
337	2.6	0.551(80)
333	3.6	0.503(46)



# Axial Charge $g_A$

$m_\pi^{\text{DWF}}$	$L$ (fm)	$g_A$
725	2.6	1.17(2)
570	2.6	1.20(4)
337	2.6	1.06(11)
333	3.6	1.24(9)

